

thm_2Ebinary_2Eneg_2Eulp
(TMaRwdtganQcTeyqe1q3NsXbSgpyPN2j6Mr)

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Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{ty_2Erealax_2Ereal}) \tag{4}$$

Definition 1 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p\ (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2E2T to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define c_2Ebool_2E21 to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (ty_2Erealax_2Ereal_REP_CLASS\ a)))$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \tag{5}$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_neg)$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (8)$$

Let $ty_2Efcpcart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcpcart\ A0\ A1) \quad (9)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1) \quad (10)$$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign\ A_27t\ A_27w \in ((ty_2Efcpcart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (11)$$

Let $ty_2Efcpcart_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efcpcart_2Efinite_image\ A0) \quad (12)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (13)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (14)$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \quad (15)$$

Let $c_2Efcpcart_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efcpcart_2Edimindex\ A_27a \in (ty_2Eenum_2Eenum)^{(ty_2Ebool_2Eitself\ A_27a)} \quad (16)$$

Definition 8 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (17)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (18)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (19)$$

Definition 12 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num)$

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40))))$

Definition 14 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C)))$

Definition 16 We define $c_2Efcf_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40) (A_27a^{ty_2Enum_2Enum}))$

Let $c_2Efcf_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcf_2Edest_cart \\ & A_27a A_27b \in ((A_27a^{(ty_2Efcf_2Efinite_image A_27b)})^{(ty_2Efcf_2Ecart A_27a A_27b)}) \end{aligned} \quad (20)$$

Definition 17 We define $c_2Efcf_2Efcf_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcf_2Ecart A_27a A_27b)$

Definition 18 We define c_2Efcf_2EFCF to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap (c_2Efcf_2Efcf_index)))$

Definition 19 We define $c_2Ewords_2Eword_1comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcf_2Ecart 2 A_27a)$.

Definition 20 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Let $c_2Ebinary_ieee_2Efloat_Sign_fupd : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign_fupd \\ & A_27t A_27w \in (((ty_2Ebinary_ieee_2Efloat A_27t A_27w)^{(ty_2Ebinary_ieee_2Efloat A_27t A_27w)})^{(ty_2Efcf_2Ecart A_27t A_27w)}) \end{aligned} \quad (21)$$

Definition 21 We define $c_Ebinary_ieee_Efloat_negate$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. \lambda V0x \in (ty_Ebinary_ieee_Efloat_plus_min$

Let $c_Ebinary_ieee_Efloat_plus_min : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Efloat_plus_min\ A_27t\ A_27w \in ((ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \quad (22)$$

Let $c_Eenum_EZERO_REP : \iota$ be given. Assume the following.

$$c_Eenum_EZERO_REP \in \omega \quad (23)$$

Definition 22 We define c_Eenum_E0 to be $(ap\ c_Eenum_EABS_num\ c_Eenum_EZERO_REP)$.

Definition 23 We define $c_Earithmetic_EZERO$ to be c_Eenum_E0 .

Let $c_Earithmetic_E2B : \iota$ be given. Assume the following.

$$c_Earithmetic_E2B \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum}) \quad (24)$$

Definition 24 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_Eenum_Eenum.(ap\ (ap\ c_Earithmetic_E2B\ n))$

Definition 25 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_Eenum_Eenum.V0x$.

Let $c_Ereal_Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_Ereal_of_num \in (ty_Erealax_Ereal^{ty_Eenum_Eenum}) \quad (25)$$

Let $c_Ereal_Epow : \iota$ be given. Assume the following.

$$c_Ereal_Epow \in ((ty_Erealax_Ereal^{ty_Eenum_Eenum})^{ty_Erealax_Ereal}) \quad (26)$$

Let $c_Ebinary_ieee_Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Efloat_Significand\ A_27t\ A_27w \in ((ty_Efcpx_Ecart\ 2\ A_27t)^{(ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)}) \quad (27)$$

Let $c_Earithmetic_EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_EEXP \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum}) \quad (28)$$

Definition 26 We define c_Ebool_ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (c_Ebool_ECOND\ t1\ t2\ t3)))$

Definition 27 We define c_Ebit_ESBIT to be $\lambda V0b \in 2. \lambda V1n \in ty_Eenum_Eenum.(ap\ (ap\ (ap\ (c_Ebool_ECOND\ b\ n\ n))))$

Let $c_Esum_num_ESUM : \iota$ be given. Assume the following.

$$c_Esum_num_ESUM \in ((ty_Eenum_Eenum^{(ty_Eenum_Eenum^{ty_Eenum_Eenum})})^{ty_Eenum_Eenum}) \quad (29)$$

Definition 28 We define c_Ewords_2Ew2n to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap\ (ap\ c_Erealax_2Ereal_inv : \iota$ be given. Assume the following.

$$c_Erealax_2Ereal_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)} \quad (30)$$

Definition 29 We define $c_Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_Erealax_2Ereal_ABS$ Let $c_Erealax_2Ereal_mul : \iota$ be given. Assume the following.

$$c_Erealax_2Ereal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)} \quad (31)$$

Definition 30 We define $c_Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 31 We define $c_Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 32 We define $c_Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_Earithmic_2E$ Let $c_Erealax_2Ereal_add : \iota$ be given. Assume the following.

$$c_Erealax_2Ereal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)} \quad (32)$$

Definition 33 We define $c_Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_Ewords_2EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ewords_2EINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (33)$$

Let $c_Ebinary_2Eieee_2Efloat_2EExponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_2Eieee_2Efloat_2EExponent\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ A_27w)^{(ty_2Ebinary_2Eieee_2Efloat\ A_27t\ A_27w)}) \quad (34)$$

Let $c_Earithmic_2EDIV : \iota$ be given. Assume the following.

$$c_Earithmic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})_{ty_2Enum_2Enum}) \quad (35)$$

Definition 34 We define $c_Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_Earithmic_2E_2D : \iota$ be given. Assume the following.

$$c_Earithmic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})_{ty_2Enum_2Enum}) \quad (36)$$

Let $c_Earithmic_2EMOD : \iota$ be given. Assume the following.

$$c_Earithmic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})_{ty_2Enum_2Enum}) \quad (37)$$

Definition 35 We define `c_2Ebit_2EMOD_2EXP` to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 36 We define `c_2Ebit_2EBITS` to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 37 We define `c_2Ebit_2EBIT` to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 38 We define `c_2Ewords_2En2w` to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcpc_2EFC$

Definition 39 We define `c_2Ebinary_2ieee_2Efloat_2to_2real` to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinar$

Let `c_2Ebinary_2ieee_2Eulp` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_2ieee_2Eulp \\ & A_27t\ A_27w \in (ty_2Erealax_2Ereal^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in (ty_2Ebinary_2ieee_2Efloat_2to_2real\ A_27a\ A_27b).((ap\ (c_2Ebinary_2ieee_2Efloat_2to_2real \\ & A_27a\ A_27b)\ (ap\ (c_2Ebinary_2ieee_2Efloat_2negate\ A_27a\ A_27b) \\ & V0x)) = (ap\ c_2Erealax_2Ereal_2neg\ (ap\ (c_2Ebinary_2ieee_2Efloat_2to_2real \\ & A_27a\ A_27b)\ V0x)))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow (\\ & (ap\ (c_2Ebinary_2ieee_2Eulp\ A_27t\ A_27w)\ (c_2Ebool_2Ethe_2value \\ & (ty_2Epair_2Eprod\ A_27t\ A_27w)))) = (ap\ (c_2Ebinary_2ieee_2Efloat_2to_2real \\ & A_27t\ A_27w)\ (ap\ (c_2Ebinary_2ieee_2Efloat_2plus_2min\ A_27t \\ & A_27w)\ (c_2Ebool_2Ethe_2value\ (ty_2Epair_2Eprod\ A_27t\ A_27w)))))) \end{aligned} \quad (40)$$

Assume the following.

$$True \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (42)$$

Theorem 1

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow (\\ & (ap\ c_2Erealax_2Ereal_2neg\ (ap\ (c_2Ebinary_2ieee_2Eulp\ A_27t \\ & A_27w)\ (c_2Ebool_2Ethe_2value\ (ty_2Epair_2Eprod\ A_27t\ A_27w)))) = \\ & (ap\ (c_2Ebinary_2ieee_2Efloat_2to_2real\ A_27t\ A_27w)\ (ap\ (c_2Ebinary_2ieee_2Efloat_2negate \\ & A_27t\ A_27w)\ (ap\ (c_2Ebinary_2ieee_2Efloat_2plus_2min\ A_27t \\ & A_27w)\ (c_2Ebool_2Ethe_2value\ (ty_2Epair_2Eprod\ A_27t\ A_27w)))))) \end{aligned}$$