

thm_2Ebinary_2Eieee_2ERound_roundTiesToEven_is_minus_zero (TMbMi2SZGkse7Vd8X9da6FWvWQtQZ5LfWSB)

October 26, 2020

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (1)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (2)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (3)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (4)$$

Let $c_2Ebinary_2Eieee_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_2Eieee_2Elargest\ A_27t\ A_27w \in (ty_2Erealax_2Ereal\ (ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))) \quad (5)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (6)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (7)$$

Definition 1 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p\ (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealx_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (8)$$

Let $c_2Erealx_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (9)$$

Let $c_2Erealx_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_ABS_CLASS \in (ty_2Erealx_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})} \quad (10)$$

Definition 6 We define $c_2Erealx_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty$

Definition 7 We define $c_2Erealx_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.(ap c_2Erealx_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (13)$$

Definition 8 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal)^{ty_2Enum_2Enum} \quad (14)$$

Let $c_2Erealx_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (15)$$

Definition 9 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$

Definition 10 We define c_Ebool_EF to be $(ap (c_Ebool_E21) 2) (\lambda V0t \in 2.V0t)$.

Definition 11 We define $c_Emin_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow Q)$ of type ι .

Definition 12 We define c_Ebool_E7E to be $(\lambda V0t \in 2.(ap (ap c_Emin_3D_3D_3E V0t) c_Ebool_E21))$

Definition 13 We define $c_Ereal_Ereal_lte$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Definition 14 We define $c_Ebool_E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E21) 2) (\lambda V2t \in 2.V2t)))$

Definition 15 We define c_Ebool_ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (ap (ap (c_Ebool_ECOND) 2) 2) 2) 2) 2))$

Definition 16 We define c_Ereal_Eabs to be $\lambda V0x \in ty_Erealax_Ereal.(ap (ap (ap (c_Ebool_ECOND) 2) 2) 2) 2)$

Let $c_Efcf_Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Efcf_Edimindex A_27a \in (ty_Eenum_Eenum^{(ty_Ebool_Eitself A_27a)}) \quad (16)$$

Definition 17 We define $c_Earithmic_EZERO$ to be c_Eenum_E0 .

Let $c_Eenum_EERP_num : \iota$ be given. Assume the following.

$$c_Eenum_EERP_num \in (\omega^{ty_Eenum_Eenum}) \quad (17)$$

Let $c_Eenum_EESUC_REP : \iota$ be given. Assume the following.

$$c_Eenum_EESUC_REP \in (\omega^{\omega}) \quad (18)$$

Definition 18 We define c_Eenum_EESUC to be $\lambda V0m \in ty_Eenum_Eenum.(ap c_Eenum_EABS_num)$

Let $c_Earithmic_E2B : \iota$ be given. Assume the following.

$$c_Earithmic_E2B \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum}) \quad (19)$$

Definition 19 We define $c_Earithmic_EBIT2$ to be $\lambda V0n \in ty_Eenum_Eenum.(ap (ap c_Earithmic_E2B) 2)$

Definition 20 We define $c_Earithmic_ENUMERAL$ to be $\lambda V0x \in ty_Eenum_Eenum.V0x$.

Let $c_Ereal_Epow : \iota$ be given. Assume the following.

$$c_Ereal_Epow \in ((ty_Erealax_Ereal^{ty_Eenum_Eenum})^{ty_Erealax_Ereal}) \quad (20)$$

Let $ty_Efcf_Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_Efcf_Ecart A0 A1) \quad (21)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1) \quad (22)$$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (23)$$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Efinite_image\ A0) \quad (24)$$

Definition 21 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E40$

Definition 22 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 23 We define $c_2Ebool_2E3F_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ c_2Ebool_2E2F_5C$

Definition 24 We define $c_2Efc_2Efinite_index$ to be $\lambda A_27a : \iota. (ap\ (c_2Emin_2E40\ (A_27a^{ty_2Enum_2Enum}$

Let $c_2Efc_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efc_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efc_2Efinite_image\ A_27b)})^{(ty_2Efc_2Ecart\ A_27a\ A_27b)}) \quad (25)$$

Definition 25 We define $c_2Efc_2Efc_index$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in (ty_2Efc_2Ecart\ A_27a$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (26)$$

Definition 26 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2. \lambda V1n \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_2Ebo$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (27)$$

Definition 27 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a). (ap\ (ap\ c$

Let $c_2Erealax_2Etreax_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (28)$$

Definition 28 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (29)$$

Definition 29 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 30 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 31 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2EBIT1$

Let $c_2Erealax_2Etreal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (30)$$

Definition 32 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_2Ewords_2EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)})(ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (31)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent\ A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (32)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (33)$$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign\ A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (34)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (35)$$

Definition 33 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (36)$$

Let $c_Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (37)$$

Definition 34 We define $c_Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 35 We define c_Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum. \lambda V$

Definition 36 We define c_Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap$

Definition 37 We define c_EfcP_2EFCP to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0g \in (A_27a^{ty_2Enum_2Enum}). (ap$

Definition 38 We define c_Ewords_2En2w to be $\lambda A_27a : \iota. \lambda V0n \in ty_2Enum_2Enum. (ap (c_EfcP_2EFCP$

Definition 39 We define $c_Ebinary_ieee_2Efloat_to_real$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. \lambda V0x \in (ty_2Ebinary$

Let $ty_2Ebinary_ieee_2Efloat_value : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Efloat_value \quad (38)$$

Let $c_Ebinary_ieee_2Efloat : \iota$ be given. Assume the following.

$$c_Ebinary_ieee_2Efloat \in (ty_2Ebinary_ieee_2Efloat_value^{ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal} \quad (39)$$

Let $c_Ebinary_ieee_2ENaN : \iota$ be given. Assume the following.

$$c_Ebinary_ieee_2ENaN \in ty_2Ebinary_ieee_2Efloat_value \quad (40)$$

Let $c_Ebinary_ieee_2EInfinity : \iota$ be given. Assume the following.

$$c_Ebinary_ieee_2EInfinity \in ty_2Ebinary_ieee_2Efloat_value \quad (41)$$

Let $c_Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_Ewords_2EUINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)})^{ty_2Enum_2Enum} \quad (42)$$

Definition 40 We define $c_Ewords_2Eword_T$ to be $\lambda A_27a : \iota. (ap (c_Ewords_2En2w\ A_27a) (ap (c_Ew$

Definition 41 We define $c_Ebinary_ieee_2Efloat_value$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. \lambda V0x \in (ty_2Ebinary$

Let $c_Ebinary_ieee_2Efloat_value_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})^{ty_2Ebinary_ieee_2Efloat_value} \quad (43)$$

Definition 42 We define $c_Ebinary_ieee_Efloat_is_finite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebina$

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_EABS_prod \\ A_27a\ A_27b \in ((ty_Epair_Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (44)$$

Definition 43 We define $c_Epair_E_EC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $c_Epred_set_EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epred_set_EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_Epair_Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (45)$$

Definition 44 We define $c_Ecombin_EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 45 We define $c_Ereal_Ereal_sub$ to be $\lambda V0x \in ty_Erealx_Ereal.\lambda V1y \in ty_Erealx_Ereal$

Definition 46 We define c_Ebool_EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Definition 47 We define $c_Ebinary_ieee_Eis_closest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (2^{(ty_Ebina$

Definition 48 We define $c_Ebinary_ieee_Eclosest_such$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (2^{(ty_Ebina$

Definition 49 We define $c_Ebinary_ieee_Eclosest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (c_Ebinary_ieee_Eclose$

Let c_Ebina

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebina_ieee_Efloat_top \\ A_27t\ A_27w \in ((ty_Ebina_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))} \end{aligned} \quad (46)$$

Definition 50 We define $c_Ereal_Ereal_gt$ to be $\lambda V0x \in ty_Erealx_Ereal.\lambda V1y \in ty_Erealx_Ereal$

Let $c_Ebina_ieee_Efloat_bottom : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebina_ieee_Efloat_bottom \\ A_27t\ A_27w \in ((ty_Ebina_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))} \end{aligned} \quad (47)$$

Definition 51 We define c_Ebool_ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27$

Let $c_Ebina_ieee_Efloat_minus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebina_ieee_Efloat_minus_infinity \\ A_27t\ A_27w \in ((ty_Ebina_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))} \end{aligned} \quad (48)$$

Definition 52 We define $c_Ereal_Ereal_ge$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Let $c_Ebinary_ieee_Efloat_plus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Efloat_plus_infinity\ A_27t\ A_27w \in ((ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \quad (49)$$

Let $c_Ebinary_ieee_Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Ethreshold\ A_27t\ A_27w \in (ty_Erealax_Ereal^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \quad (50)$$

Definition 53 We define $c_Ewords_Eword_lsb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efcpr_Ecart\ 2\ A_27a).(ap$

Let $ty_Ebinary_ieee_ERounding : \iota$ be given. Assume the following.

$$nonempty\ ty_Ebinary_ieee_ERounding \quad (51)$$

Let $c_Ebinary_ieee_ERounding2num : \iota$ be given. Assume the following.

$$c_Ebinary_ieee_ERounding2num \in (ty_Enum_Enum^{ty_Ebinary_ieee_ERounding}) \quad (52)$$

Definition 54 We define $c_Ebinary_ieee_ERounding_CASE$ to be $\lambda A_27a : \iota.\lambda V0x \in ty_Ebinary_ieee_E$

Definition 55 We define $c_Ebinary_ieee_ERound$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_Ebinary_iee$

Let $c_Ebinary_ieee_Efloat_plus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Efloat_plus_zero\ A_27t\ A_27w \in ((ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \quad (53)$$

Let $c_Ebinary_ieee_Efloat_minus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Efloat_minus_zero\ A_27t\ A_27w \in ((ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \quad (54)$$

Definition 56 We define $c_Ebinary_ieee_Efloat_is_zero$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebina$

Definition 57 We define $c_Ebinary_ieee_Efloat_round$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_Ebina$

Definition 58 We define $c_Ebinary_ieee_Efloat_is_infinite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebina$

Definition 59 We define $c_Ebinary_ieee_Efloat_is_subnormal$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_E$

Definition 60 We define $c_Ebinary_ieee_Efloat_is_normal$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebin$

Assume the following.

$$\begin{aligned}
& \forall A.27t.nonempty A.27t \Rightarrow \forall A.27w.nonempty A.27w \Rightarrow (\\
& \quad (p (ap (c.2Ebinary_ieee.2Efloat_is_zero A.27t A.27w) (ap (\\
& \quad \quad c.2Ebinary_ieee.2Efloat_plus_zero A.27t A.27w) (c.2Ebool.2Ethe_value \\
& \quad \quad (ty_2Epair_2Eprod A.27t A.27w)))))) \wedge ((p (ap (c.2Ebinary_ieee.2Efloat_is_zero \\
& \quad \quad A.27t A.27w) (ap (c.2Ebinary_ieee.2Efloat_minus_zero A.27t \\
& \quad \quad A.27w) (c.2Ebool.2Ethe_value (ty_2Epair_2Eprod A.27t A.27w)))))) \wedge \\
& \quad ((p (ap (c.2Ebinary_ieee.2Efloat_is_finite A.27t A.27w) (\\
& \quad \quad ap (c.2Ebinary_ieee.2Efloat_plus_zero A.27t A.27w) (c.2Ebool.2Ethe_value \\
& \quad \quad (ty_2Epair_2Eprod A.27t A.27w)))))) \wedge ((p (ap (c.2Ebinary_ieee.2Efloat_is_finite \\
& \quad \quad A.27t A.27w) (ap (c.2Ebinary_ieee.2Efloat_minus_zero A.27t \\
& \quad \quad A.27w) (c.2Ebool.2Ethe_value (ty_2Epair_2Eprod A.27t A.27w)))))) \wedge \\
& \quad ((p (ap (c.2Ebinary_ieee.2Efloat_is_integral A.27t A.27w) (\\
& \quad \quad ap (c.2Ebinary_ieee.2Efloat_plus_zero A.27t A.27w) (c.2Ebool.2Ethe_value \\
& \quad \quad (ty_2Epair_2Eprod A.27t A.27w)))))) \wedge ((p (ap (c.2Ebinary_ieee.2Efloat_is_integral \\
& \quad \quad A.27t A.27w) (ap (c.2Ebinary_ieee.2Efloat_minus_zero A.27t \\
& \quad \quad A.27w) (c.2Ebool.2Ethe_value (ty_2Epair_2Eprod A.27t A.27w)))))) \wedge \\
& \quad ((\neg(p (ap (c.2Ebinary_ieee.2Efloat_is_nan A.27t A.27w) (ap \\
& \quad \quad (c.2Ebinary_ieee.2Efloat_plus_zero A.27t A.27w) (c.2Ebool.2Ethe_value \\
& \quad \quad (ty_2Epair_2Eprod A.27t A.27w)))))) \wedge ((\neg(p (ap (c.2Ebinary_ieee.2Efloat_is_nan \\
& \quad \quad A.27t A.27w) (ap (c.2Ebinary_ieee.2Efloat_minus_zero A.27t \\
& \quad \quad A.27w) (c.2Ebool.2Ethe_value (ty_2Epair_2Eprod A.27t A.27w)))))) \wedge \\
& \quad ((\neg(p (ap (c.2Ebinary_ieee.2Efloat_is_normal A.27t A.27w) (\\
& \quad \quad ap (c.2Ebinary_ieee.2Efloat_plus_zero A.27t A.27w) (c.2Ebool.2Ethe_value \\
& \quad \quad (ty_2Epair_2Eprod A.27t A.27w)))))) \wedge ((\neg(p (ap (c.2Ebinary_ieee.2Efloat_is_normal \\
& \quad \quad A.27t A.27w) (ap (c.2Ebinary_ieee.2Efloat_minus_zero A.27t \\
& \quad \quad A.27w) (c.2Ebool.2Ethe_value (ty_2Epair_2Eprod A.27t A.27w)))))) \wedge \\
& \quad ((\neg(p (ap (c.2Ebinary_ieee.2Efloat_is_subnormal A.27t A.27w) (\\
& \quad \quad ap (c.2Ebinary_ieee.2Efloat_plus_zero A.27t A.27w) (c.2Ebool.2Ethe_value \\
& \quad \quad (ty_2Epair_2Eprod A.27t A.27w)))))) \wedge ((\neg(p (ap (c.2Ebinary_ieee.2Efloat_is_subnormal \\
& \quad \quad A.27t A.27w) (ap (c.2Ebinary_ieee.2Efloat_minus_zero A.27t \\
& \quad \quad A.27w) (c.2Ebool.2Ethe_value (ty_2Epair_2Eprod A.27t A.27w)))))) \wedge \\
& \quad ((\neg(p (ap (c.2Ebinary_ieee.2Efloat_is_infinite A.27t A.27w) (\\
& \quad \quad ap (c.2Ebinary_ieee.2Efloat_plus_zero A.27t A.27w) (c.2Ebool.2Ethe_value \\
& \quad \quad (ty_2Epair_2Eprod A.27t A.27w)))))) \wedge (\neg(p (ap (c.2Ebinary_ieee.2Efloat_is_infinite \\
& \quad \quad A.27t A.27w) (ap (c.2Ebinary_ieee.2Efloat_minus_zero A.27t \\
& \quad \quad A.27w) (c.2Ebool.2Ethe_value (ty_2Epair_2Eprod A.27t A.27w)))))))))
\end{aligned}$$

(57)

Assume the following.

$$\begin{aligned}
& \forall A.27t.nonempty\ A.27t \Rightarrow \forall A.27w.nonempty\ A.27w \Rightarrow (\\
& \quad \forall V0x \in ty_2Erealax_2Ereal.((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& \quad (ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) \\
& \quad (ap\ c_2Ereal_2Eabs\ V0x)))\ (ap\ (c_2Ebinary_ieee_2Eulp\ A.27t\ A.27w) \\
& \quad (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A.27t\ A.27w)))))) \Rightarrow \\
& (((ap\ (ap\ (c_2Ebinary_ieee_2ERound\ A.27t\ A.27w)\ c_2Ebinary_ieee_2ERoundTiesToEven) \\
& \quad V0x) = (ap\ (c_2Ebinary_ieee_2Efloat_plus_zero\ A.27t\ A.27w) \\
& \quad (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A.27t\ A.27w)))) \vee (\\
& (ap\ (ap\ (c_2Ebinary_ieee_2ERound\ A.27t\ A.27w)\ c_2Ebinary_ieee_2ERoundTiesToEven) \\
& \quad V0x) = (ap\ (c_2Ebinary_ieee_2Efloat_minus_zero\ A.27t\ A.27w) \\
& \quad (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A.27t\ A.27w))))))))) \\
& \hspace{15em} (58)
\end{aligned}$$

Assume the following.

$$True \tag{59}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{60}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{61}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{62}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (A.27b^{A.27a}).(\forall V1x \in A.27a.((ap\ (ap\ (c_2Ebool_2ELET \\
& \quad A.27a\ A.27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \tag{63}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\
& \quad True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\
& \quad (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \tag{64}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& \quad ((\neg False) \Leftrightarrow True))) \tag{65}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow \\
& \quad True)) \tag{66}
\end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (67)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (68)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (69)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (70)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (71)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27)\ V5y_27)))))) \quad (72)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \quad (73)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (74)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (75)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (76)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (77)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (78)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (79)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (80)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (81)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (82)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (83)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (84)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (85)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (86)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (87)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (88)$$

Theorem 1

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow (\\ & \quad \forall V0x \in ty_2Erealax_2Ereal. ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\ & \quad (ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Ereal_2Ereal_of_num \\ & \quad (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))))) \\ & \quad (ap\ c_2Ereal_2Eabs\ V0x)))\ (ap\ (c_2Ebinary_ieee_2Eulp\ A_27t\ A_27w) \\ & \quad (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_27t\ A_27w)))))) \Rightarrow \\ & ((ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_round\ A_27t\ A_27w)\ c_2Ebinary_ieee_2ERoundTiesToEven) \\ & \quad c_2Ebool_2ET)\ V0x) = (ap\ (c_2Ebinary_ieee_2Efloat_minus_zero \\ & \quad A_27t\ A_27w)\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_27t \\ & \quad A_27w)))))) \end{aligned}$$