

thm\_2Ebinary\_2iee\_2Ethreshold  
(TMVSZpgv9CWZqjpYtq9zVSCTGWJsv7R7Feu)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{3}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{4}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{5}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{6}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{7}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (8)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (9)$$

**Definition 5** We define  $c\_2Emin\_2E.40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap\ P\ x))$  **then** (the  $(\lambda x.x \in A \wedge p\ x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 6** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E.40\ (ty\_2Erealax\_2Ereal\ a)))$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (10)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (11)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}} \quad (12)$$

**Definition 7** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 8** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_neg)$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (13)$$

**Definition 9** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 10** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (14)$$

Let  $c\_2Ewords\_2EINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_MAX\ A\_27a \in (ty\_2Enum\_2Enum)^{ty\_2Ebool\_2Eitself\ A\_27a} \quad (15)$$

Let  $c\_Ewords\_EUINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_Ewords\_EUINT\_MAX\ A\_27a \in ( ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)} ) \quad (16)$$

Let  $c\_Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (17)$$

Let  $c\_Ebinary\_ieee\_2Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_2Ethreshold\ A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \quad (18)$$

**Definition 11** We define  $c\_Ebool\_2EF$  to be  $(ap\ (c\_Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 12** We define  $c\_Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 13** We define  $c\_Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_Ebool\_2E\_7E))$

**Definition 14** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (19)$$

**Definition 15** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (20)$$

**Definition 16** We define  $c\_Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_Earithmetic\_2E\_2B\ V0n))$

Let  $c\_2Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (21)$$

**Definition 17** We define  $c\_2Erealax\_2Einvm$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS)$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (22)$$

**Definition 18** We define  $c\_2Emarker\_2Eunint$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.V0x$ .



Assume the following.

$$\begin{aligned}
& \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow ( \\
& \quad (ap\ (c\_2Ebinary\_ieee\_2Ethreshold\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_value \\
& \quad \quad (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))) = (ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\
& \quad (ap\ (ap\ c\_2Ereal\_2E\_2F\ (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))))) \\
& \quad \quad (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Ewords\_2EUINT\_MAX\ A\_27w) \\
& \quad \quad (c\_2Ebool\_2Ethe\_value\ A\_27w)))\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad \quad \quad (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))\ (ap\ (ap \\
& \quad \quad c\_2Ereal\_2Epow\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO))))\ (ap\ (c\_2Ewords\_2EINT\_MAX \\
& \quad \quad A\_27w)\ (c\_2Ebool\_2Ethe\_value\ A\_27w))))\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub \\
& \quad \quad (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL\ ( \\
& \quad \quad \quad ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO))))\ (ap\ c\_2Erealax\_2Einv \\
& \quad (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad \quad (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO))))\ (ap\ c\_2Enum\_2ESUC \\
& \quad \quad \quad (ap\ (c\_2Efcfcp\_2Edimindex\ A\_27t)\ (c\_2Ebool\_2Ethe\_value\ A\_27t))))))))) \\
& \hspace{15em} (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\neg((ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))) \\
& \quad \quad V0n) = (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)))))) \\
& \hspace{15em} (31)
\end{aligned}$$

Assume the following.

$$True \hspace{15em} (32)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \\
& \hspace{15em} (33)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \hspace{15em} (34)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow \\
& \quad \quad True)) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\
& \quad \quad A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\
& \quad ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \\
& \hspace{15em} (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\
& (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\
& V5y\_27)))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a. (\forall V1t2 \in \\
& A\_27a. ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap \\
& (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V2t1)\ V3t2) = V3t2))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealx\_2Ereal. ((ap\ c\_2Erealx\_2Einv\ V0x) = \\
& (ap\ (ap\ c\_2Ereal\_2E\_2F\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))\ V0x)))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ V0x))\ V1n) = \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ V0x) \\
& V1n))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealx\_2Ereal. (\forall V1y \in ty\_2Erealx\_2Ereal. \\
& (\forall V2z \in ty\_2Erealx\_2Ereal. ((ap\ (ap\ c\_2Erealx\_2Ereal\_mul \\
& (ap\ (ap\ c\_2Ereal\_2E\_2F\ V0x)\ V1y))\ V2z) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND \\
& ty\_2Erealx\_2Ereal)\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ ty\_2Erealx\_2Ereal) \\
& V1y)\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)))\ (ap\ (ap\ c\_2Erealx\_2Ereal\_mul \\
& (ap\ (c\_2Emarker\_2Euint\ ty\_2Erealx\_2Ereal)\ (ap\ (ap\ c\_2Ereal\_2E\_2F \\
& V0x)\ V1y)))\ V2z))\ (ap\ (ap\ c\_2Ereal\_2E\_2F\ (ap\ (ap\ c\_2Erealx\_2Ereal\_mul \\
& V0x)\ V2z))\ V1y))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (c\_2Ewords\_2Edimword\ A\_27a) \\
& (c\_2Ebool\_2Ethe\_value\ A\_27a)) = (ap\ (ap\ c\_2Earithmetic\_2EEXP \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))) \\
& (ap\ (c\_2EfcP\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_value\ A\_27a))))
\end{aligned} \tag{43}$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow ( \\ & (ap\ (c\_2Ebinary\_ieee\_2Ethreshold\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_value \\ & (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))) = (ap\ (ap\ c\_2Ereal\_2E\_2F\ (ap\ ( \\ & ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ ( \\ & ap\ c\_2Earithmetic\_2EEXP\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2 \\ & c\_2Earithmetic\_2EZERO)))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Ewords\_2EUINT\_MAX \\ & A\_27w)\ (c\_2Ebool\_2Ethe\_value\ A\_27w)))\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))\ (ap\ ( \\ & ap\ c\_2Ereal\_2Ereal\_sub\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))\ (ap\ (ap \\ & c\_2Ereal\_2E\_2F\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\ & (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap \\ & c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))\ (ap\ (c\_2Ewords\_2Edimword \\ & A\_27t)\ (c\_2Ebool\_2Ethe\_value\ A\_27t))))))\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\ & (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap \\ & c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))\ (ap\ (c\_2Ewords\_2EINT\_MAX \\ & A\_27w)\ (c\_2Ebool\_2Ethe\_value\ A\_27w)))))) \end{aligned}$$