

thm_2Ebinary_2ieee_2Eulp_2lt_2ULP
(TMbHw3hhBtVC1VoHjsaBcfhLrtSyJvm9Rmm)

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Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (1)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (2)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (3)$$

Let $c_2Ebinary_2ieee_2Eulp : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_2ieee_2Eulp\ A_27t\ A_27w \in (ty_2Erealax_2Ereal^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \quad (4)$$

Let $c_2Ebool_2Ethe_2Evalue : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_2Evalue\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (5)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_2Eo\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_2Eo\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 3 We define c_2Ebool_2E2T to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a})))$

Definition 5 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2))\ (\lambda V2t \in 2.V2t)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}})$$
 (6)

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum$$
 (7)

Let $c_2Efcf_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efcf_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)})$$
 (8)

Let $c_2Ewords_2EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)})$$
 (9)

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})$$
 (10)

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
 (11)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
 (12)

Definition 7 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 8 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum})$$
 (13)

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega})$$
 (14)

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ ($

Definition 10 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 11 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})_{ty_2Erealax_2Ereal}) \quad (16)$$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Efinite_image\ A0) \quad (17)$$

Definition 12 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E))$

Definition 14 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge P\ x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 15 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ V0P))))$

Definition 16 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 17 We define $c_2Ebool_2E3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_2Ebool_2E2F_5C\ V0P))))$

Definition 18 We define $c_2Efc_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_2Emin_2E40\ (A_27a^{ty_2Enum_2Enum})))$

Let $ty_2Efc_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efc_2Ecart\ A0\ A1) \quad (18)$$

Let $c_2Efc_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efc_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efc_2Efinite_image\ A_27b)})_{(ty_2Efc_2Ecart\ A_27a\ A_27b)}) \quad (19)$$

Definition 19 We define $c_2Efc_2Efc_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efc_2Ecart\ A_27a\ A_27b)$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})_{ty_2Enum_2Enum}) \quad (20)$$

Definition 20 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(c_2Ebool_2ECOND\ V1t1\ V2t2))))$

Definition 21 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ V0b))))$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})_{ty_2Enum_2Enum}) \quad (21)$$

Definition 22 We define c_Ewords_Ew2n to be $\lambda A.27a : \iota. \lambda V0w \in (ty_EfcP_Ecart\ 2\ A.27a).(ap\ (ap\ c_Earithmic_EBIT1\ ty_Ew2n\ w))$

Definition 23 We define $c_Earithmic_EBIT1$ to be $\lambda V0n \in ty_Eenum_Eenum.(ap\ (ap\ c_Earithmic_EBIT1\ ty_Eenum_Eenum\ n))$

Let $c_Earithmic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmic_EDIV \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum}) \quad (22)$$

Definition 24 We define $c_Ebit_EDIV_EXP$ to be $\lambda V0x \in ty_Eenum_Eenum.\lambda V1n \in ty_Eenum_Eenum$

Let $c_Earithmic_E_2D : \iota$ be given. Assume the following.

$$c_Earithmic_E_2D \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum}) \quad (23)$$

Let $c_Earithmic_EMOD : \iota$ be given. Assume the following.

$$c_Earithmic_EMOD \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum}) \quad (24)$$

Definition 25 We define $c_Ebit_EMOD_EXP$ to be $\lambda V0x \in ty_Eenum_Eenum.\lambda V1n \in ty_Eenum_Eenum$

Definition 26 We define c_Ebit_EBITS to be $\lambda V0h \in ty_Eenum_Eenum.\lambda V1l \in ty_Eenum_Eenum.\lambda V2n \in ty_Eenum_Eenum$

Definition 27 We define c_Ebit_EBIT to be $\lambda V0b \in ty_Eenum_Eenum.\lambda V1n \in ty_Eenum_Eenum.(ap\ (ap\ c_Earithmic_EBIT1\ ty_Eenum_Eenum\ n))$

Definition 28 We define c_EfcP_EFCP to be $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0g \in (A.27a^{ty_Eenum_Eenum}).(ap\ (ap\ c_Earithmic_EBIT1\ ty_Eenum_Eenum\ g)))$

Definition 29 We define c_Ewords_En2w to be $\lambda A.27a : \iota.\lambda V0n \in ty_Eenum_Eenum.(ap\ (c_EfcP_EFCP\ ty_Eenum_Eenum\ n))$

Let $ty_Ehreal_Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_Ehreal_Ehreal \quad (25)$$

Let $c_Erealax_Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_REP_CLASS \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}\ ty_Erealax_Ereal_REP_CLASS)) \quad (26)$$

Definition 30 We define $c_Erealax_Ereal_REP$ to be $\lambda V0a \in ty_Erealax_Ereal_REP.(ap\ (c_Emin_E40\ ty_Erealax_Ereal_REP\ a))$

Let $c_Erealax_Etreal_inv : \iota$ be given. Assume the following.

$$c_Erealax_Etreal_inv \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}) \quad (27)$$

Let $c_Erealax_Etreal_eq : \iota$ be given. Assume the following.

$$c_Erealax_Etreal_eq \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}\ ty_Erealax_Etreal_eq)) \quad (28)$$

Let $c_Erealax_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_ABS_CLASS \in (ty_Erealax_Ereal)^{(2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}\ ty_Erealax_Ereal)} \quad (29)$$

Definition 31 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 32 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (30)$$

Definition 33 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 34 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 35 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 36 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 37 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a}))\ A_27a))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (31)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (32)$$

Definition 38 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair_2Epair)$

Definition 39 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap\ (c_2Ebool_2E_2I\ 2^{A_27a}))$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (33)$$

Definition 40 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1M \in (A_27b)^{(A_27a)}$

Definition 41 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in (A_27a)^{(A_27a)}$

Definition 42 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27b)^{(A_27a)}$

Definition 43 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27b)^{(A_27a)}$

Definition 44 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27b)^{(A_27a)}$

Definition 45 We define $c_2Ebinary_ieee_2EULP$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. (ap (ap (c_2Erelation_2EWFRL$

Definition 46 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Let $c_2Erealax_2Etreal_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (34)$$

Definition 47 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 48 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Eenum_2Eenum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (35)$$

Definition 49 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Efcpc_2Ecart\ 2\ A_27a).$

Definition 50 We define $c_2Ewords_2Eword_2msb$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Efcpc_2Ecart\ 2\ A_27a).$

Definition 51 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A_27a}). (\lambda V1x \in A_27b.$

Definition 52 We define $c_2Ewords_2Enzcv$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Efcpc_2Ecart\ 2\ A_27a). \lambda V1b \in ($

Definition 53 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a}$

Definition 54 We define $c_2Ewords_2Eword_2ls$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Efcpc_2Ecart\ 2\ A_27a). \lambda V1b \in ($

Assume the following.

$$\begin{aligned} & \forall A_27t. nonempty\ A_27t \Rightarrow \forall A_27w. nonempty\ A_27w \Rightarrow (\\ & (ap (c_2Ebinary_ieee_2Eulp\ A_27t\ A_27w)\ (c_2Ebool_2Ethe_value \\ & (ty_2Epair_2Eprod\ A_27t\ A_27w))) = (ap (c_2Ebinary_ieee_2EULP \\ & A_27t\ A_27w)\ (ap (ap (c_2Epair_2E_2C\ (ty_2Efcpc_2Ecart\ 2\ A_27w) \\ & (ty_2Ebool_2Eitself\ A_27t))\ (ap (c_2Ewords_2En2w\ A_27w)\ c_2Eenum_2E0)) \\ & (c_2Ebool_2Ethe_value\ A_27t)))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27t. nonempty\ A_27t \Rightarrow (\\ & \forall V0e1 \in (ty_2Efcpc_2Ecart\ 2\ A_27a). (\forall V1e2 \in (ty_2Efcpc_2Ecart \\ & 2\ A_27a). ((\neg (V1e2 = (ap (c_2Ewords_2En2w\ A_27a)\ c_2Eenum_2E0))) \Rightarrow \\ & ((p (ap (ap\ c_2Ereal_2Ereal_lte\ (ap (c_2Ebinary_ieee_2EULP \\ & A_27t\ A_27a)\ (ap (ap (c_2Epair_2E_2C\ (ty_2Efcpc_2Ecart\ 2\ A_27a) \\ & (ty_2Ebool_2Eitself\ A_27t))\ V0e1)\ (c_2Ebool_2Ethe_value\ A_27t)))) \\ & (ap (c_2Ebinary_ieee_2EULP\ A_27t\ A_27a)\ (ap (ap (c_2Epair_2E_2C \\ & (ty_2Efcpc_2Ecart\ 2\ A_27a)\ (ty_2Ebool_2Eitself\ A_27t))\ V1e2) \\ & (c_2Ebool_2Ethe_value\ A_27t)))))) \Leftrightarrow (p (ap (ap (c_2Ewords_2Eword_2ls \\ & A_27a)\ V0e1)\ V1e2)))))) \end{aligned} \quad (37)$$

Assume the following.

$$True \quad (38)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (41)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (43)$$

Assume the following.

$$(\forall V0x \in ty_2Erealx_2Ereal.(p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (44)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0w \in (ty_2Efcpx_2Ecart 2 A_27a).(p (ap (ap (c_2Ewords_2Eword_ls A_27a) (ap (c_2Ewords_2En2w A_27a) c_2Enum_2E0)) V0w))) \quad (45)$$

Theorem 1

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow (\forall V0e \in (ty_2Efcpx_2Ecart 2 A_27w).(p (ap (ap c_2Ereal_2Ereal_lte (ap (c_2Ebinary_iee_2Eulp A_27t A_27w) (c_2Ebool_2Ethe_value (ty_2Epair_2Eprod A_27t A_27w)))) (ap (c_2Ebinary_iee_2EULP A_27t A_27w) (ap (ap (c_2Epair_2E_2C (ty_2Efcpx_2Ecart 2 A_27w) (ty_2Ebool_2Eitself A_27t)) V0e) (c_2Ebool_2Ethe_value A_27t))))))$$