

thm_2Ebinary_2Ezero_properties (TMYK3gfm1HHgr797MUbpR67NUpsJVqKbaXk)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let `ty_2Erealax_2Ereal` : ι be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let `ty_2Ebool_2Eitself` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \tag{2}$$

Let `c_2Ebool_2Ethe_value` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \tag{3}$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{4}$$

Let `c_2EfcP_2Edimindex` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2EfcP_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \tag{5}$$

Let `c_2Enum_2EZERO_REP` : ι be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{6}$$

Let `c_2Enum_2EABS_num` : ι be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{7}$$

Definition 5 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 6 We define $c_2Arithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (9)$$

Definition 7 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ c_2Enum_2ESUC_REP)$.

Let $c_2Arithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Arithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (10)$$

Definition 8 We define $c_2Arithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Arithmetic_2E_2B\ n)\ c_2Enum_2E0)$.

Definition 9 We define $c_2Arithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (11)$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal)^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal} \quad (12)$$

Let $ty_2EfcP_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2EfcP_2Ecart\ A0\ A1) \quad (13)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1) \quad (14)$$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (15)$$

Let $ty_2EfcP_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Efinite_image\ A0) \quad (16)$$

Definition 10 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_7E$

Definition 12 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21) 2) (\lambda V2t \in 2.$

Definition 13 We define $c_Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge P x)$ of type $\iota \Rightarrow \iota$).

Definition 14 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_Emin_2E_40$

Definition 15 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_Ebool_2E_2F_5C$

Definition 17 We define $c_Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_Emin_2E_40 (A_27a^{ty_2Enum_2Enum}$

Let $c_Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Efcp_2Edest_cart \\ & A_27a A_27b \in ((A_27a^{(ty_2Efc_2Efinite_image A_27b)})^{(ty_2Efc_2Ecart A_27a A_27b)}) \end{aligned} \quad (17)$$

Definition 18 We define $c_Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efc_2Ecart A_27a$

Let $c_Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 19 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda V3t3 \in 2.$

Definition 20 We define c_Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_Ebool_2E_7E$

Let $c_Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (19)$$

Definition 21 We define c_Ewords_2Ew2n to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart 2 A_27a).(ap (ap c_Ebool_2E_7E$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal \quad (20)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod \\ & A0 A1) \end{aligned} \quad (21)$$

Let $c_Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal_REP_CLASS}) \quad (22)$$

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (t$
Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (23)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (24)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}})) \quad (25)$$

Definition 23 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 24 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (26)$$

Definition 25 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 26 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.($

Definition 27 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (27)$$

Definition 28 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Ewords_2EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewords_2EINT_MAX A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (28)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent A_27t A_27w \in ((ty_2EfcP_2Ecart 2 A_27w)^{(ty_2Ebinary_ieee_2Efloat A_27t A_27w)}) \quad (29)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (30)$$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign \\ & A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (31)$$

Let $c_2Erealax_2Etreax_neg : \iota$ be given. Assume the following.

$$\begin{aligned} & c_2Erealax_2Etreax_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (32)$$

Definition 29 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (33)$$

Definition 30 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (34)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (35)$$

Definition 31 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 32 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 33 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 34 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 35 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Efc_2EFCP$

Definition 36 We define $c_2Ebinary_ieee_2Efloat_to_real$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinar$

Let $ty_2Ebinary_ieee_2Efloat_value : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Efloat_value \quad (36)$$

Let $c_2Ebinary_ieee_2Efloat : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Efloat \in (ty_2Ebinary_ieee_2Efloat_value^{ty_2Erealax_2Ereal}) \quad (37)$$

Let $c_2Ebinary_ieee_2ENaN : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2ENaN \in ty_2Ebinary_ieee_2Efloat_value \quad (38)$$

Let $c_2Ebinary_ieee_2EInfinity : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2EInfinity \in ty_2Ebinary_ieee_2Efloat_value \quad (39)$$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (40)$$

Definition 37 We define $c_2Ewords_2Eword_T$ to be $\lambda A_27a : \iota.(ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ (c_2Eword_T\ A_27a)\ (c_2Eword_T\ A_27a)))$

Definition 38 We define $c_2Ebinary_ieee_2Efloat_value$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_ieee_2Efloat_value)\ (c_2Efloat_value\ A_27w\ V0x)$

Let $c_2Ebinary_ieee_2Efloat_value_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebinary_ieee_2Efloat_value_CASE\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})_{ty_2Ebinary_ieee_2Efloat_value} \quad (41)$$

Definition 39 We define $c_2Ebinary_ieee_2Efloat_is_nan$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_ieee_2Efloat_value)\ (c_2Efloat_is_nan\ A_27w\ V0x)$

Definition 40 We define $c_2Ebinary_ieee_2Efloat_is_infinite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_ieee_2Efloat_value)\ (c_2Efloat_is_infinite\ A_27w\ V0x)$

Definition 41 We define $c_2Ebinary_ieee_2Efloat_is_normal$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_ieee_2Efloat_value)\ (c_2Efloat_is_normal\ A_27w\ V0x)$

Definition 42 We define $c_2Ebinary_ieee_2Efloat_is_subnormal$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_ieee_2Efloat_value)\ (c_2Efloat_is_subnormal\ A_27w\ V0x)$

Definition 43 We define $c_2Ebinary_ieee_2Efloat_is_zero$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_ieee_2Efloat_value)\ (c_2Efloat_is_zero\ A_27w\ V0x)$

Definition 44 We define $c_2Ebinary_ieee_2Efloat_is_finite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_ieee_2Efloat_value)\ (c_2Efloat_is_finite\ A_27w\ V0x)$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (42)$$

Definition 45 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.\ (c_2Ereal_lt\ T1\ T2)$

Definition 46 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.\ (c_2Ereal_lte\ x\ y)$

Definition 47 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECONV)\ (c_2Eabs)\ (c_2Eabs)))\ x)$

Definition 48 We define $c_2Ebinary_ieee_2Eis_integral$ to be $\lambda V0r \in ty_2Erealax_2Ereal.(ap\ (c_2Ebool_2ECONV)\ (c_2Eis_integral\ r))$

Definition 49 We define $c_2Ebinary_ieee_2Efloat_is_integral$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_ieee_2Efloat_value)\ (c_2Efloat_is_integral\ A_27w\ V0x)$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (43)$$

Let $c_2Ebinary_ieee_2Efloat_Significand_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27u.nonempty\ A_27u \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand_fupd \\ & A_27t\ A_27u\ A_27w \in (((ty_2Ebinary_ieee_2Efloat\ A_27u\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})) \end{aligned} \quad (44)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow \forall A_27x.nonempty\ A_27x \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent_fupd\ A_27t\ A_27w \\ & A_27w\ A_27x \in (((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27x)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})) \end{aligned} \quad (45)$$

Let $c_2Ebinary_ieee_2Efloat_Sign_fupd : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign_fupd\ A_27t\ A_27w \\ & A_27t\ A_27w \in (((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})) \end{aligned} \quad (46)$$

Definition 50 We define $c_2Ewords_2Eword_1comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcpr_2Ecart\ 2\ A_27a).$

Definition 51 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 52 We define $c_2Ebinary_ieee_2Efloat_negate$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w).$

Let $c_2Ebinary_ieee_2Efloat_minus_min : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_min\ A_27t\ A_27w \\ & A_27t\ A_27w \in (((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (47)$$

Let $c_2Ebinary_ieee_2Efloat_plus_min : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_min\ A_27t\ A_27w \\ & A_27t\ A_27w \in (((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (48)$$

Let $c_2Ebinary_ieee_2Efloat_minus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_zero\ A_27t\ A_27w \\ & A_27t\ A_27w \in (((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (49)$$

Let $c_2Ebinary_ieee_2Efloat_plus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_zero \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (50)$$

Definition 53 We define $c_2Ewords_2Eword_msb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap$

Definition 54 We define $c_2Ebinary_ieee_2Efloat_is_signalling$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Efloat$

Definition 55 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b$

Let $ty_2Ebinary_ieee_2Efp_op : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efp_op \\ & A0\ A1) \end{aligned} \quad (51)$$

Definition 56 We define $c_2Ebinary_ieee_2Efloat_some_qnan$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0fp_op \in (ty_2Efloat$

Let $c_2Ebinary_ieee_2Efloat_minus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_infinity \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (52)$$

Let $c_2Ebinary_ieee_2Efloat_plus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_infinity \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (53)$$

Definition 57 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2$

Definition 58 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27a^{A_27c}).$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (54)$$

Definition 59 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Assume the following.

$$\begin{aligned}
& \forall A_{.27t}.nonempty\ A_{.27t} \Rightarrow \forall A_{.27u}.nonempty\ A_{.27u} \Rightarrow \forall A_{.27w}. \\
& \quad nonempty\ A_{.27w} \Rightarrow \forall A_{.27x}.nonempty\ A_{.27x} \Rightarrow ((\forall V0f0 \in \\
& \quad ((ty_2EfcP_2Ecart\ 2\ A_{.27x})^{(ty_2EfcP_2Ecart\ 2\ A_{.27w})}).(\forall V1f \in \\
& \quad (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_{.27t}\ A_{.27x})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent_fupd \\
& \quad A_{.27t}\ A_{.27w}\ A_{.27x})\ V0f0)\ V1f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_{.27t}\ A_{.27w})\ V1f)))) \wedge ((\forall V2f0 \in ((ty_2EfcP_2Ecart\ 2\ A_{.27u})^{(ty_2EfcP_2Ecart\ 2\ A_{.27t})}). \\
& \quad (\forall V3f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Sign\ A_{.27u}\ A_{.27w})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Significand_fupd \\
& \quad A_{.27t}\ A_{.27u}\ A_{.27w})\ V2f0)\ V3f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_{.27t}\ A_{.27w})\ V3f)))) \wedge ((\forall V4f0 \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)} \\
& \quad (\forall V5f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Exponent\ A_{.27t}\ A_{.27w})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Sign_fupd \\
& \quad A_{.27t}\ A_{.27w})\ V4f0)\ V5f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent \\
& \quad A_{.27t}\ A_{.27w})\ V5f)))) \wedge ((\forall V6f0 \in ((ty_2EfcP_2Ecart\ 2\ A_{.27u})^{(ty_2EfcP_2Ecart\ 2\ A_{.27t})}). \\
& \quad (\forall V7f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Exponent\ A_{.27u}\ A_{.27w})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Significand_fupd \\
& \quad A_{.27t}\ A_{.27u}\ A_{.27w})\ V6f0)\ V7f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent \\
& \quad A_{.27t}\ A_{.27w})\ V7f)))) \wedge ((\forall V8f0 \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)} \\
& \quad (\forall V9f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27t}\ A_{.27w})\ (ap\ (ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Sign_fupd\ A_{.27t}\ A_{.27w})\ V8f0)\ V9f)) = \\
& \quad (ap\ (c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27t}\ A_{.27w})\ V9f)))) \wedge \\
& \quad ((\forall V10f0 \in ((ty_2EfcP_2Ecart\ 2\ A_{.27x})^{(ty_2EfcP_2Ecart\ 2\ A_{.27w})}). \\
& \quad (\forall V11f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ \\
& \quad (c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27t}\ A_{.27x})\ (ap\ (ap \\
& \quad (c_2EbinaRy_ieee_2Efloat_Exponent_fupd\ A_{.27t}\ A_{.27w}\ A_{.27x}) \\
& \quad V10f0)\ V11f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27t} \\
& \quad A_{.27w})\ V11f)))) \wedge ((\forall V12f0 \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)} \\
& \quad (\forall V13f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ \\
& \quad (c_2EbinaRy_ieee_2Efloat_Sign\ A_{.27t}\ A_{.27w})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Sign_fupd \\
& \quad A_{.27t}\ A_{.27w})\ V12f0)\ V13f)) = (ap\ V12f0\ (ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_{.27t}\ A_{.27w})\ V13f)))) \wedge ((\forall V14f0 \in ((ty_2EfcP_2Ecart\ 2 \\
& \quad A_{.27x})^{(ty_2EfcP_2Ecart\ 2\ A_{.27w})}).(\forall V15f \in (ty_2EbinaRy_ieee_2Efloat \\
& \quad A_{.27t}\ A_{.27w}).((ap\ (c_2EbinaRy_ieee_2Efloat_Exponent\ A_{.27t} \\
& \quad A_{.27x})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent_fupd\ A_{.27t} \\
& \quad A_{.27w}\ A_{.27x})\ V14f0)\ V15f)) = (ap\ V14f0\ (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent \\
& \quad A_{.27t}\ A_{.27w})\ V15f)))) \wedge ((\forall V16f0 \in ((ty_2EfcP_2Ecart\ 2\ A_{.27u})^{(ty_2EfcP_2Ecart\ 2\ A_{.27t})}). \\
& \quad (\forall V17f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ \\
& \quad (c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27u}\ A_{.27w})\ (ap\ (ap \\
& \quad (c_2EbinaRy_ieee_2Efloat_Significand_fupd\ A_{.27t}\ A_{.27u}\ A_{.27w}) \\
& \quad V16f0)\ V17f)) = (ap\ V16f0\ (ap\ (c_2EbinaRy_ieee_2Efloat_Significand \\
& \quad A_{.27t}\ A_{.27w})\ V17f)))))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& \forall A.27t.nonempty\ A.27t \Rightarrow \forall A.27u.nonempty\ A.27u \Rightarrow \forall A.27v. \\
& nonempty\ A.27v \Rightarrow \forall A.27w.nonempty\ A.27w \Rightarrow \forall A.27x.nonempty \\
& A.27x \Rightarrow \forall A.27y.nonempty\ A.27y \Rightarrow ((\forall V0g \in ((ty_2Efc_2Ecart \\
& 2\ ty_2Eone_2Eone)^{(ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)}), (\forall V1f0 \in \\
& ((ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)}), \\
& (\forall V2f \in (ty_2Ebinary_ieee_2Efloat\ A.27t\ A.27w)).((ap\ (\\
& ap\ (c_2Ebinary_ieee_2Efloat_Sign_fupd\ A.27t\ A.27w)\ V1f0) \\
& (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_Sign_fupd\ A.27t\ A.27w)\ V0g) \\
& V2f))) = (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_Sign_fupd\ A.27t\ A.27w) \\
& (ap\ (ap\ (c_2Ecombin_2Eo\ (ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)\ (\\
& ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)\ (ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)) \\
& V1f0)\ V0g))\ V2f)))) \wedge ((\forall V3g \in ((ty_2Efc_2Ecart\ 2\ A.27x)^{(ty_2Efc_2Ecart\ 2\ A.27w)}), \\
& (\forall V4f0 \in ((ty_2Efc_2Ecart\ 2\ A.27y)^{(ty_2Efc_2Ecart\ 2\ A.27x)}), \\
& (\forall V5f \in (ty_2Ebinary_ieee_2Efloat\ A.27t\ A.27w)).((ap\ (\\
& ap\ (c_2Ebinary_ieee_2Efloat_Exponent_fupd\ A.27t\ A.27x\ A.27y) \\
& V4f0)\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_Exponent_fupd\ A.27t \\
& A.27w\ A.27x)\ V3g)\ V5f))) = (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_Exponent_fupd \\
& A.27t\ A.27w\ A.27y)\ (ap\ (ap\ (c_2Ecombin_2Eo\ (ty_2Efc_2Ecart\ 2 \\
& A.27w)\ (ty_2Efc_2Ecart\ 2\ A.27y)\ (ty_2Efc_2Ecart\ 2\ A.27x)) \\
& V4f0)\ V3g))\ V5f)))) \wedge ((\forall V6g \in ((ty_2Efc_2Ecart\ 2\ A.27u)^{(ty_2Efc_2Ecart\ 2\ A.27t)}), \\
& (\forall V7f0 \in ((ty_2Efc_2Ecart\ 2\ A.27v)^{(ty_2Efc_2Ecart\ 2\ A.27u)}), \\
& (\forall V8f \in (ty_2Ebinary_ieee_2Efloat\ A.27t\ A.27w)).((ap\ (\\
& ap\ (c_2Ebinary_ieee_2Efloat_Significand_fupd\ A.27u\ A.27v \\
& A.27w)\ V7f0)\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_Significand_fupd \\
& A.27t\ A.27u\ A.27w)\ V6g)\ V8f))) = (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_Significand_fupd \\
& A.27t\ A.27v\ A.27w)\ (ap\ (ap\ (c_2Ecombin_2Eo\ (ty_2Efc_2Ecart\ 2 \\
& A.27t)\ (ty_2Efc_2Ecart\ 2\ A.27v)\ (ty_2Efc_2Ecart\ 2\ A.27u)) \\
& V7f0)\ V6g))\ V8f))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0a \in ty_2Erealax_2Ereal. \\
& (\forall V1f \in (A.27a^{ty_2Erealax_2Ereal}).(\forall V2v \in A.27a. \\
& (\forall V3v1 \in A.27a.((ap\ (ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_value_CASE \\
& A.27a)\ (ap\ c_2Ebinary_ieee_2Efloat\ V0a))\ V1f)\ V2v)\ V3v1) = (ap \\
& V1f\ V0a)))))) \wedge ((\forall V4f \in (A.27a^{ty_2Erealax_2Ereal}).(\forall V5v \in \\
& A.27a.(\forall V6v1 \in A.27a.((ap\ (ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_value_CASE \\
& A.27a)\ c_2Ebinary_ieee_2EInfinity)\ V4f)\ V5v)\ V6v1) = V5v)))) \wedge \\
& (\forall V7f \in (A.27a^{ty_2Erealax_2Ereal}).(\forall V8v \in A.27a. \\
& (\forall V9v1 \in A.27a.((ap\ (ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_value_CASE \\
& A.27a)\ c_2Ebinary_ieee_2ENaN)\ V7f)\ V8v)\ V9v1) = V9v1))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow (\\
& (ap\ (c_2Ebinary_ieee_2Efloat_plus_zero\ A_27t\ A_27w)\ (c_2Ebool_2Ethe_value \\
& (ty_2Epair_2Eprod\ A_27t\ A_27w))) = (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_Sign_fupd \\
& A_27t\ A_27w)\ (ap\ (c_2Ecombin_2EK\ (ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone) \\
& (ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone))\ (ap\ (c_2Ewords_2En2w\ ty_2Eone_2Eone) \\
& c_2Enum_2E0)))\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_Exponent_fupd \\
& A_27t\ A_27w\ A_27w)\ (ap\ (c_2Ecombin_2EK\ (ty_2Efc_2Ecart\ 2\ A_27w) \\
& (ty_2Efc_2Ecart\ 2\ A_27w))\ (ap\ (c_2Ewords_2En2w\ A_27w)\ c_2Enum_2E0)))) \\
& (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_Significand_fupd\ A_27t \\
& A_27t\ A_27w)\ (ap\ (c_2Ecombin_2EK\ (ty_2Efc_2Ecart\ 2\ A_27t)\ (ty_2Efc_2Ecart \\
& 2\ A_27t))\ (ap\ (c_2Ewords_2En2w\ A_27t)\ c_2Enum_2E0))))\ (c_2Ebool_2EARB \\
& (ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow (\\
& (ap\ (c_2Ebinary_ieee_2Efloat_minus_zero\ A_27t\ A_27w)\ (c_2Ebool_2Ethe_value \\
& (ty_2Epair_2Eprod\ A_27t\ A_27w))) = (ap\ (c_2Ebinary_ieee_2Efloat_negate \\
& A_27t\ A_27w)\ (ap\ (c_2Ebinary_ieee_2Efloat_plus_zero\ A_27t \\
& A_27w)\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_27t\ A_27w))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27t}. \\
& nonempty\ A_{.27t} \Rightarrow \forall A_{.27w}.nonempty\ A_{.27w} \Rightarrow (((ap\ (c_2Ebinary_ieee_2Efloat_value \\
& \quad A_{.27t}\ A_{.27w})\ (ap\ (c_2Ebinary_ieee_2Efloat_plus_infinity \\
& \quad A_{.27t}\ A_{.27w})\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_{.27t} \\
& \quad A_{.27w})))) = c_2Ebinary_ieee_2EInfinity) \wedge (((ap\ (c_2Ebinary_ieee_2Efloat_value \\
& \quad A_{.27t}\ A_{.27w})\ (ap\ (c_2Ebinary_ieee_2Efloat_minus_infinity \\
& \quad A_{.27t}\ A_{.27w})\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_{.27t} \\
& \quad A_{.27w})))) = c_2Ebinary_ieee_2EInfinity) \wedge ((\forall V0fp_op \in \\
& \quad (ty_2Ebinary_ieee_2Efp_op\ A_{.27a}\ A_{.27b}).((ap\ (c_2Ebinary_ieee_2Efloat_value \\
& \quad A_{.27a}\ A_{.27b})\ (ap\ (c_2Ebinary_ieee_2Efloat_some_qnan\ A_{.27a} \\
& \quad A_{.27b})\ V0fp_op)) = c_2Ebinary_ieee_2ENaN) \wedge (((ap\ (c_2Ebinary_ieee_2Efloat_value \\
& \quad A_{.27t}\ A_{.27w})\ (ap\ (c_2Ebinary_ieee_2Efloat_plus_zero\ A_{.27t} \\
& \quad A_{.27w})\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_{.27t}\ A_{.27w})))) = \\
& \quad (ap\ c_2Ebinary_ieee_2EFloat\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))) \wedge \\
& \quad (((ap\ (c_2Ebinary_ieee_2Efloat_value\ A_{.27t}\ A_{.27w})\ (ap\ (c_2Ebinary_ieee_2Efloat_minus_zero \\
& \quad A_{.27t}\ A_{.27w})\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_{.27t} \\
& \quad A_{.27w})))) = (ap\ c_2Ebinary_ieee_2EFloat\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))) \wedge (((ap\ (c_2Ebinary_ieee_2Efloat_value\ A_{.27t} \\
& \quad A_{.27w})\ (ap\ (c_2Ebinary_ieee_2Efloat_plus_min\ A_{.27t}\ A_{.27w}) \\
& \quad (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_{.27t}\ A_{.27w})))) = (\\
& \quad ap\ c_2Ebinary_ieee_2EFloat\ (ap\ (ap\ c_2Ereal_2E2F\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) \\
& \quad (ap\ (ap\ c_2Ereal_2Epow\ (ap\ c_2Ereal_2Ereal_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))))\ (ap\ (ap \\
& \quad c_2Earithmetic_2E2B\ (ap\ (c_2Ewords_2EINT_MAX\ A_{.27w})\ (c_2Ebool_2Ethe_value \\
& \quad A_{.27w})))\ (ap\ (c_2Efc_2Edimindex\ A_{.27t})\ (c_2Ebool_2Ethe_value \\
& \quad A_{.27t})))))) \wedge ((ap\ (c_2Ebinary_ieee_2Efloat_value\ A_{.27t}\ A_{.27w}) \\
& \quad (ap\ (c_2Ebinary_ieee_2Efloat_minus_min\ A_{.27t}\ A_{.27w})\ (c_2Ebool_2Ethe_value \\
& \quad (ty_2Epair_2Eprod\ A_{.27t}\ A_{.27w})))) = (ap\ c_2Ebinary_ieee_2EFloat \\
& \quad (ap\ (ap\ c_2Ereal_2E2F\ (ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) \\
& \quad (ap\ (ap\ c_2Ereal_2Epow\ (ap\ c_2Ereal_2Ereal_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))))\ (ap\ (ap \\
& \quad c_2Earithmetic_2E2B\ (ap\ (c_2Ewords_2EINT_MAX\ A_{.27w})\ (c_2Ebool_2Ethe_value \\
& \quad A_{.27w})))\ (ap\ (c_2Efc_2Edimindex\ A_{.27t})\ (c_2Ebool_2Ethe_value \\
& \quad A_{.27t})))))))))
\end{aligned} \tag{60}$$

Assume the following.

$$True \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\
& \quad (p\ V0t)) \Leftrightarrow (p\ V0t))))))
\end{aligned} \tag{62}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (63)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (64)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (65)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (66)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))) \quad (67)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0a \in A_27a.(\exists V1x \in A_27a.(V1x = V0a))) \quad (68)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.((ap (ap (c_2Ecombin_2EK A_27a A_27b) V0x) V1y) = V0x))) \quad (69)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c.nonempty A_27c \Rightarrow \forall A_27d.nonempty A_27d \Rightarrow \forall A_27e.nonempty A_27e \Rightarrow \forall A_27f.nonempty A_27f \Rightarrow ((\forall V0f \in (A_27b^{A_27a}). \\ & (\forall V1v \in A_27c.((ap (ap (c_2Ecombin_2Eo A_27a A_27c A_27b) (ap (c_2Ecombin_2EK A_27c A_27b) V1v)) V0f) = (ap (c_2Ecombin_2EK A_27c A_27a) V1v)))) \wedge (\forall V2f \in (A_27e^{A_27d}).(\forall V3v \in \\ & A_27d.((ap (ap (c_2Ecombin_2Eo A_27f A_27e A_27d) V2f) (ap (c_2Ecombin_2EK A_27d A_27f) V3v)) = (ap (c_2Ecombin_2EK A_27e A_27f) (ap V2f V3v)))))) \quad (70) \end{aligned}$$

Assume the following.

$$((ap\ c_2Ereal_2Eabs\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) = \quad (71)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\ & \quad (((ap\ c_2Ereal_2Ereal_of_num\ V0n) = (ap\ c_2Ereal_2Ereal_of_num\ V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num\ V0n)) = (ap\ c_2Ereal_2Ereal_of_num\ V1m)) \Leftrightarrow ((V0n = c_2Enum_2E0) \wedge (V1m = c_2Enum_2E0))) \wedge (((ap\ c_2Ereal_2Ereal_of_num\ V0n) = (ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num\ V1m))) \Leftrightarrow ((V0n = c_2Enum_2E0) \wedge (V1m = c_2Enum_2E0))) \wedge (((ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num\ V0n)) = (ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num\ V1m))) \Leftrightarrow (V0n = V1m)))))) \end{aligned} \quad (72)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum. (\\ & \quad \forall V1n \in ty_2Enum_2Enum. (((ap\ (c_2Ewords_2En2w\ A_27a)\ V0m) = (ap\ (c_2Ewords_2En2w\ A_27a)\ V1n)) \Leftrightarrow ((ap\ (ap\ c_2Earithmetic_2EMOD\ V0m)\ (ap\ (c_2Ewords_2Edimword\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a))) = (ap\ (ap\ c_2Earithmetic_2EMOD\ V1n)\ (ap\ (c_2Ewords_2Edimword\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)))))) \end{aligned} \quad (73)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Ewords_2Eword_2comp\ A_27a)\ (ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) = (c_2Ewords_2Eword_T\ A_27a)) \end{aligned} \quad (74)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Ewords_2Eword_1comp\ A_27a)\ (ap\ (c_2Ewords_2En2w\ A_27a)\ c_2Enum_2E0)) = (c_2Ewords_2Eword_T\ A_27a)) \end{aligned} \quad (75)$$

Theorem 1

$$\begin{aligned}
& \forall A_27t.\text{nonempty } A_27t \Rightarrow \forall A_27w.\text{nonempty } A_27w \Rightarrow (\\
& (p (ap (c_2Ebinary_ieee_2Efloat_is_zero A_27t A_27w) (ap (\\
& \quad c_2Ebinary_ieee_2Efloat_plus_zero A_27t A_27w) (c_2Ebool_2Ethe_value \\
& \quad (ty_2Epair_2Eprod A_27t A_27w)))))) \wedge ((p (ap (c_2Ebinary_ieee_2Efloat_is_zero \\
& \quad A_27t A_27w) (ap (c_2Ebinary_ieee_2Efloat_minus_zero A_27t \\
& \quad A_27w) (c_2Ebool_2Ethe_value (ty_2Epair_2Eprod A_27t A_27w)))))) \wedge \\
& \quad ((p (ap (c_2Ebinary_ieee_2Efloat_is_finite A_27t A_27w) (\\
& \quad ap (c_2Ebinary_ieee_2Efloat_plus_zero A_27t A_27w) (c_2Ebool_2Ethe_value \\
& \quad (ty_2Epair_2Eprod A_27t A_27w)))))) \wedge ((p (ap (c_2Ebinary_ieee_2Efloat_is_finite \\
& \quad A_27t A_27w) (ap (c_2Ebinary_ieee_2Efloat_minus_zero A_27t \\
& \quad A_27w) (c_2Ebool_2Ethe_value (ty_2Epair_2Eprod A_27t A_27w)))))) \wedge \\
& \quad ((p (ap (c_2Ebinary_ieee_2Efloat_is_integral A_27t A_27w) \\
& \quad (ap (c_2Ebinary_ieee_2Efloat_plus_zero A_27t A_27w) (c_2Ebool_2Ethe_value \\
& \quad (ty_2Epair_2Eprod A_27t A_27w)))))) \wedge ((p (ap (c_2Ebinary_ieee_2Efloat_is_integral \\
& \quad A_27t A_27w) (ap (c_2Ebinary_ieee_2Efloat_minus_zero A_27t \\
& \quad A_27w) (c_2Ebool_2Ethe_value (ty_2Epair_2Eprod A_27t A_27w)))))) \wedge \\
& \quad ((\neg(p (ap (c_2Ebinary_ieee_2Efloat_is_nan A_27t A_27w) (ap \\
& \quad (c_2Ebinary_ieee_2Efloat_plus_zero A_27t A_27w) (c_2Ebool_2Ethe_value \\
& \quad (ty_2Epair_2Eprod A_27t A_27w)))))) \wedge ((\neg(p (ap (c_2Ebinary_ieee_2Efloat_is_nan \\
& \quad A_27t A_27w) (ap (c_2Ebinary_ieee_2Efloat_minus_zero A_27t \\
& \quad A_27w) (c_2Ebool_2Ethe_value (ty_2Epair_2Eprod A_27t A_27w)))))) \wedge \\
& \quad ((\neg(p (ap (c_2Ebinary_ieee_2Efloat_is_normal A_27t A_27w) \\
& \quad (ap (c_2Ebinary_ieee_2Efloat_plus_zero A_27t A_27w) (c_2Ebool_2Ethe_value \\
& \quad (ty_2Epair_2Eprod A_27t A_27w)))))) \wedge ((\neg(p (ap (c_2Ebinary_ieee_2Efloat_is_normal \\
& \quad A_27t A_27w) (ap (c_2Ebinary_ieee_2Efloat_minus_zero A_27t \\
& \quad A_27w) (c_2Ebool_2Ethe_value (ty_2Epair_2Eprod A_27t A_27w)))))) \wedge \\
& \quad ((\neg(p (ap (c_2Ebinary_ieee_2Efloat_is_subnormal A_27t A_27w) \\
& \quad (ap (c_2Ebinary_ieee_2Efloat_plus_zero A_27t A_27w) (c_2Ebool_2Ethe_value \\
& \quad (ty_2Epair_2Eprod A_27t A_27w)))))) \wedge ((\neg(p (ap (c_2Ebinary_ieee_2Efloat_is_subnormal \\
& \quad A_27t A_27w) (ap (c_2Ebinary_ieee_2Efloat_minus_zero A_27t \\
& \quad A_27w) (c_2Ebool_2Ethe_value (ty_2Epair_2Eprod A_27t A_27w)))))) \wedge \\
& \quad ((\neg(p (ap (c_2Ebinary_ieee_2Efloat_is_infinite A_27t A_27w) \\
& \quad (ap (c_2Ebinary_ieee_2Efloat_plus_zero A_27t A_27w) (c_2Ebool_2Ethe_value \\
& \quad (ty_2Epair_2Eprod A_27t A_27w)))))) \wedge ((\neg(p (ap (c_2Ebinary_ieee_2Efloat_is_infinite \\
& \quad A_27t A_27w) (ap (c_2Ebinary_ieee_2Efloat_minus_zero A_27t \\
& \quad A_27w) (c_2Ebool_2Ethe_value (ty_2Epair_2Eprod A_27t A_27w)))))))))
\end{aligned}$$