thm_2Ebinary__ieee_2Ezero__to__real (TMcsKJRLysDWeYh3EEjHBswPDrjazEsx96u)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40 A

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{3}$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{4}$$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \tag{5}$$

Let $ty\_2Ebinary\_\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A0\ A1) \tag{6}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{7}$$

Let $c\_2Ewords\_2EINT\_\_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_2} \tag{8}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{9}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \tag{10}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{11}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{12}$$

Definition 4 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 5 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{13}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{14}$$

Definition 6 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 7 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{15}$$

Definition 9 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 10 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{16}$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{17}$$

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \tag{18}$$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \tag{19}$$

Definition 11 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 12 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 13 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 15 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 16 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}$

Definition 17 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A\_27a : \iota.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (A\_27a^{ty\_2Enum\_2Enu}$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \tag{20}$$

Definition 18 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{21}$$

Definition 19 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 20 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Eboo}$

Let $c\_2Esum\_\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (22)$$

Definition 21 *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (23)$$

Let $ty\_2Epair\_2Eprod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (24)$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \quad (25)$$

Definition 22 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (26)$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \quad (27)$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \quad (28)$$

Definition 23 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 24 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (29)$$

Definition 25 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 26 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Definition 27 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (30)$$

Definition 28 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (31)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (32)$$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (33)$$

Definition 29 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Definition 30 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (34)$$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (35)$$

Definition 31 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 32 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 33 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Definition 34 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 35 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EFC

Definition 36 *We define* c_2Ebinary__ieee_2Efloat__to__real *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \qquad (36)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27u.nonempty\ A\_27u \Rightarrow \forall A\_27w. nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_f A\_27t\ A\_27u\ A\_27w \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27u\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \qquad (37)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow \forall A\_27x. nonempty\ A\_27x \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27 A\_27w\ A\_27x \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27x)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{((ty\_2Efc} \qquad (38)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_ A\_27t\ A\_27w \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{((ty\_2Efc} \qquad (39)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \qquad (40)$$

Definition 37 *We define* c_2Ewords_2Eword__1comp *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).$

Definition 38 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 39 *We define* c_2Ebinary__ieee_2Efloat__negate *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \qquad (41)$$

Definition 40 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ $($c_2Ebool_2E_21 $2)$ $(\lambda V2t \in$

Definition 41 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Definition 42 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 43 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 44 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Definition 45 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 46 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 47 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 48 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2E}$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{42}$$

Definition 49 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 50 *We define* c_2Enumeral_2EiDUB *to be* $\lambda V0x \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2$

Let $c\_2Enumeral\_2EiSUB : \iota$ be given. Assume the following.

$$c\_2Enumeral\_2EiSUB \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{2}) \tag{43}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{44}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{45}$$

Let $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{46}$$

Definition 51 *We define* c_2Ewords_2Eword__2comp *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).$

Let $c\_2Ewords\_2EUINT\_\_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{47}$$

Definition 52 *We define* c_2Ewords_2Eword__T *to be* $\lambda A\_27a : \iota.(ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ (ap\ (c\_2Ew$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m))) \tag{48}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1k \in ty\_2Enum\_2Enum.(\forall V2r \in ty\_2Enum\_2Enum.((\exists V3q \in ty\_2Enum\_2Enum.((V1k = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V3q)\ V0n))\ V2r)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V2r)\ V0n)))) \Rightarrow ((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n) = V2r))))) \tag{49}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n)) \Rightarrow ((ap\ (ap\ c\_2Earithmetic\_2EMOD\ \mathsf{c\_2Enum\_2E0})\ V0n) = \mathsf{c\_2Enum\_2E0}))) \tag{50}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n)) \Rightarrow (((ap\ (ap\ c\_2Earithmetic\_2EDIV\ V0n)\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0n)\ V0n) = \mathsf{c\_2Enum\_2E0})))) \tag{51}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V1y)) \Rightarrow (((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0x)\ V1y) = V0x) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0x)\ V1y)))))) \tag{52}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V0n)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0n))) \tag{53}$$

Assume the following.

$$
\begin{gathered}
\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27u.nonempty\ A\_27u \Rightarrow \forall A\_27w. \\
nonempty\ A\_27w \Rightarrow \forall A\_27x.nonempty\ A\_27x \Rightarrow ((\forall V0f0 \in \\
((ty\_2Efcp\_2Ecart\ 2\ A\_27x)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27w)}).(\forall V1f \in \\
(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign \\
A\_27t\ A\_27x)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd \\
A\_27t\ A\_27w\ A\_27x)\ V0f0)\ V1f)) = (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign \\
A\_27t\ A\_27w)\ V1f)))) \wedge ((\forall V2f0 \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27u)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27t)}). \\
(\forall V3f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\ (\\
c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27u\ A\_27w)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fu \\
A\_27t\ A\_27u\ A\_27w)\ V2f0)\ V3f)) = (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign \\
A\_27t\ A\_27w)\ V3f)))) \wedge ((\forall V4f0 \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eo} \\
(\forall V5f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\ (\\
c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27t\ A\_27w)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupc \\
A\_27t\ A\_27w)\ V4f0)\ V5f)) = (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent \\
A\_27t\ A\_27w)\ V5f)))) \wedge ((\forall V6f0 \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27u)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27t)}). \\
(\forall V7f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\ (\\
c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27u\ A\_27w)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_ \\
A\_27t\ A\_27u\ A\_27w)\ V6f0)\ V7f)) = (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent \\
A\_27t\ A\_27w)\ V7f)))) \wedge ((\forall V8f0 \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eo} \\
(\forall V9f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\ (\\
c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\ A\_27w)\ (ap\ (ap\ (\\
c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd\ A\_27t\ A\_27w)\ V8f0)\ V9f)) = \\
(ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\ A\_27w)\ V9f)))) \wedge \\
((\forall V10f0 \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27x)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27w)}). \\
(\forall V11f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap \\
(c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\ A\_27x)\ (ap\ (ap \\
(c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27t\ A\_27w\ A\_27x) \\
V10f0)\ V11f)) = (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t \\
A\_27w)\ V11f)))) \wedge ((\forall V12f0 \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone} \\
(\forall V13f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap \\
(c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27t\ A\_27w)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd \\
A\_27t\ A\_27w)\ V12f0)\ V13f)) = (ap\ V12f0\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign \\
A\_27t\ A\_27w)\ V13f))))) \wedge ((\forall V14f0 \in ((ty\_2Efcp\_2Ecart\ 2 \\
A\_27x)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27w)}).(\forall V15f \in (ty\_2Ebinary\_\_ieee\_2Efloat \\
A\_27t\ A\_27w).((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27t \\
A\_27x)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27t \\
A\_27w\ A\_27x)\ V14f0)\ V15f)) = (ap\ V14f0\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent \\
A\_27t\ A\_27w)\ V15f))))) \wedge (\forall V16f0 \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27u)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27t)}). \\
(\forall V17f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap \\
(c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27u\ A\_27w)\ (ap\ (ap \\
(c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd\ A\_27t\ A\_27u\ A\_27w) \\
V16f0)\ V17f)) = (ap\ V16f0\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand \\
A\_27t\ A\_27w)\ V17f))))))))))))
\end{gathered}
\tag{54}
$$

Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27u.nonempty\ A\_27u{\Rightarrow}\forall A\_27v.\ nonempty\ A\_27v{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}\forall A\_27x.nonempty\ A\_27x{\Rightarrow}\forall A\_27y.nonempty\ A\_27y{\Rightarrow}((\forall V0g \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)}).(\forall V1f0 \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)}).(\forall V2f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd\ A\_27t\ A\_27w)\ V1f0)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd\ A\_27t\ A\_27w)\ V0g)\ V2f)) = (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd\ A\_27t\ A\_27w)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)\ (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)\ (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone))\ V1f0)\ V0g))\ V2f)))))\wedge((\forall V3g \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27x)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27w)}).(\forall V4f0 \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27y)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27x)}).(\forall V5f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27t\ A\_27x\ A\_27y)\ V4f0)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27t\ A\_27w\ A\_27x)\ V3g)\ V5f)) = (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27t\ A\_27w\ A\_27y)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27w)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27y)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27x))\ V4f0)\ V3g))\ V5f)))))\wedge(\forall V6g \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27u)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27t)}).(\forall V7f0 \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27v)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27u)}).(\forall V8f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd\ A\_27u\ A\_27v\ A\_27w)\ V7f0)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd\ A\_27t\ A\_27u\ A\_27w)\ V6g)\ V8f)) = (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd\ A\_27t\ A\_27v\ A\_27w)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27t)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27v)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27u))\ V7f0)\ V6g))\ V8f)))))))) \tag{55}$$

Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_zero\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))) = (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd\ A\_27t\ A\_27w)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)\ (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone))\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ ty\_2Eone\_2Eone)\ \mathsf{c\_2Enum\_2E0})))\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27t\ A\_27w\ A\_27w)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27w)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27w))\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27w)\ \mathsf{c\_2Enum\_2E0})))\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd\ A\_27t\ A\_27t\ A\_27w)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27t)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27t))\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27t)\ \mathsf{c\_2Enum\_2E0})))\ (c\_2Ebool\_2EARB\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)))))) \tag{56}$$

Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow ((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_zero\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))) = (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_negate}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_zero\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))) \tag{57}$$

Assume the following.

$$True \tag{58}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{59}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{60}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{61}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27a\ A\_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \tag{62}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{63}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{66}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{67}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{68}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{69}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg (p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{70}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V2t1)\ V3t2) = V3t2)))) \tag{71}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ A\_27a\ A\_27b)\ V0x)\ V1y) = V0x))) \tag{72}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{73}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow \forall A\_27e.nonempty\ A\_27e \Rightarrow \forall A\_27f.nonempty\ A\_27f \Rightarrow ((\forall V0f \in (A\_27b^{A\_27a}).\ (\forall V1v \in A\_27c.((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27c\ A\_27b)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ A\_27c\ A\_27b)\ V1v))\ V0f) = (ap\ (\mathsf{c\_2Ecombin\_2EK}\ A\_27c\ A\_27a)\ V1v)))) \wedge (\forall V2f \in (A\_27e^{A\_27d}).(\forall V3v \in A\_27d.((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27f\ A\_27e\ A\_27d)\ V2f)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ A\_27d\ A\_27f)\ V3v)) = (ap\ (\mathsf{c\_2Ecombin\_2EK}\ A\_27e\ A\_27f)\ (ap\ V2f\ V3v)))))) \tag{74}$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$\mathsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in$$
$$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0}))\wedge$$
$$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in ty\_2Enum\_2Enum.$$
$$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($$
$$ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V7m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$\mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in$$
$$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V11m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$$
$$\mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V12n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$$
$$(ap\ c\_2Earithmetic\_2EEXP\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in$$
$$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \mathsf{c\_2Enum\_2E0}) =$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge$$
$$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$\mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in ty\_2Enum\_2Enum.($$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) =$$
$$\mathsf{c\_2Enum\_2E0})\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$$
$$(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge$$
$$((\forall V20n \in ty\_2Enum\_2Enum.((\mathsf{c\_2Enum\_2E0} = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V20n))\Leftrightarrow(V20n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in ty\_2Enum\_2Enum.$$
$$(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V21n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge$$
$$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V23n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V24n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V25n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}$$
$$\mathsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V28n))\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V29n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$\mathsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO}) \Leftrightarrow False) \wedge (((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO}) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)) \Leftrightarrow (V0n = V1m)))))))))))) \tag{76}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))))))))))) \tag{77}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.(\forall V1b \in 2.(\forall V2n \in ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.(((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ V1b)\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0x) = \mathsf{c\_2Earithmetic\_2EZERO}) \wedge (((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ V2n)\ \mathsf{c\_2Earithmetic\_2EZERO}) = V2n) \wedge (((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2n))\ \mathsf{c\_2Earithmetic\_2EZERO}) = (ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ V2n)) \wedge (((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3m)) = (ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ V2n)\ V3m))) \wedge (((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ V2n)\ V3m))) \wedge (((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ V2n)\ V3m))) \wedge (((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V3m)) = (ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ V2n)\ V3m))) \wedge (((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2n))\ \mathsf{c\_2Earithmetic\_2EZERO}) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2n)) \wedge (((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ V2n)\ V3m))) \wedge (((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3m)) = (ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ V2n)\ V3m))) \wedge (((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V3m)) = (ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ V2n)\ V3m))) \wedge ((ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2EF})\ V2n)\ V3m))))))))))))))))) \tag{78}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)\ V1m)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB\ \mathsf{c\_2Ebool\_2ET})\ V0n)\ V1m)))\ \mathsf{c\_2Enum\_2E0})))) \tag{79}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ c\_2Earithmetic\_2EEVEN\ \mathsf{c\_2Earithmetic\_2EZERO}))\wedge ((p\ (ap\ c\_2Earithmetic\_2EEVEN\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))\wedge ((\neg(p\ (ap\ c\_2Earithmetic\_2EEVEN\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))))\wedge ((\neg(p\ (ap\ c\_2Earithmetic\_2EODD\ \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\neg(p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))))\wedge (p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))))))))) \tag{80}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V0x) = V0x)) \tag{81}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = V0x)) \tag{82}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) = V0x)) \tag{83}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{84}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{85}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2a \in ty\_2Enum\_2Enum.(\forall V3y \in ty\_2Erealax\_2Ereal.(((ap\ (ap\ c\_2Ereal\_2Epow\ V0x)\ \mathsf{c\_2Enum\_2E0}) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge(((ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1n))) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\wedge(((ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1n))) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\wedge(((ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2a)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2a))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n))))\wedge(((ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2a))))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}))\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)))\ \mathsf{c\_2Erealax\_2Ereal\_\_neg})\ (\lambda V4x \in ty\_2Erealax\_2Ereal.V4x))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2a))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)))))\wedge((ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ V0x)\ V3y))\ V1n) = (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ (ap\ c\_2Ereal\_2Epow\ V0x)\ V1n))\ (ap\ (ap\ c\_2Ereal\_2Epow\ V3y)\ V1n)))))))))) \quad (86)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.((((ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V0n) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V1m))\Leftrightarrow(V0n = V1m))\wedge((((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V0n)) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V1m))\Leftrightarrow((V0n = \mathsf{c\_2Enum\_2E0})\wedge(V1m = \mathsf{c\_2Enum\_2E0})))\wedge((((ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V0n) = (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V1m)))\Leftrightarrow((V0n = \mathsf{c\_2Enum\_2E0})\wedge(V1m = \mathsf{c\_2Enum\_2E0})))\wedge(((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V0n)) = (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V1m)))\Leftrightarrow(V0n = V1m))))))) \quad (87)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))) \quad (88)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.((ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V0n)) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0n)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))) \tag{89}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V0m) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V1n)) \Leftrightarrow ((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0m)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1n)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))))) \tag{90}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0})) = \mathsf{c\_2Enum\_2E0}) \tag{91}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.((ap\ (\mathsf{c\_2Ewords\_2Eword\_\_2comp}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V0n)) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0n)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))))) \tag{92}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (\mathsf{c\_2Ewords\_2Eword\_\_2comp}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) = (\mathsf{c\_2Ewords\_2Eword\_\_T}\ A\_27a)) \tag{93}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (\mathsf{c\_2Ewords\_2Eword\_\_1comp}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0})) = (\mathsf{c\_2Ewords\_2Eword\_\_T}\ A\_27a)) \tag{94}$$

Assume the following.

$$((ap\ (c\_2Ewords\_2Edimword\ ty\_2Eone\_2Eone)\ (c\_2Ebool\_2Ethe\_\_value\ ty\_2Eone\_2Eone)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))) \tag{95}$$

Theorem 1

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow (((ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_to\_\_real}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_zero\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)))) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \wedge ((ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_to\_\_real}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_zero\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)))) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))$$