

thm_2Ebit_2EADD__BIT__SUC
 (TMHFe8bSsFotrgRtVG7SVAujhmetC8Am4Bm)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota))$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y) \text{ of type } \iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c_2Emin_2E_40 A) (c_2Emin_2E_3D A))))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \quad (1)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (2)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (3)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (4)$$

Definition 4 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 5 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (6)$$

Definition 6 We define c_2Ebool_2ET to be $(ap \ (ap \ (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap \ (ap \ (c_2Emin_2E_3D \ (2^{A_27a})) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x)))$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap \ c_2Enum_2EABS_num \ (m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum} \rightarrow ty_2Enum_2Enum) \quad (7)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap \ (ap \ c_2Earithmetic_2E_2B \ (n)))$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$

Definition 11 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap \ (ap \ c_2Earithmetic_2E_2B \ (n)))$

Let $c_2Earithmetic_2EEEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEEXP \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum} \rightarrow ty_2Enum_2Enum) \quad (8)$$

Definition 12 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2EEEXP \ (x \ div \ n))$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum} \rightarrow ty_2Enum_2Enum) \quad (9)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum} \rightarrow ty_2Enum_2Enum) \quad (10)$$

Definition 13 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2EEEXP \ (x \ mod \ n))$

Definition 14 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V2t \in 2.(c_2Ebool_2ET \ (h \ bits \ l))$

Definition 15 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap \ (c_2Ebool_2ET \ (b \ bits \ n)))$

Definition 16 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o \ (p \ P \Rightarrow p \ Q) \ of \ type \ \iota$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap \ (c_2Ebool_2E_21 \ 2) \ (\lambda V2t \in 2.(c_2Ebool_2ET \ (t1 \ and \ t2)))) \ (\lambda V3t \in 2.(c_2Ebool_2ET \ (t1 \ or \ t2))))$

Definition 18 We define $c_2Ebool_2EBOUNDED$ to be $(\lambda V0v \in 2.c_2Ebool_2ET).$

Definition 19 We define c_2Ebool_2EF to be $(ap \ (c_2Ebool_2E_21 \ 2) \ (\lambda V0t \in 2.V0t)).$

Definition 20 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap \ (ap \ c_2Emin_2E_3D_3D_3E \ V0t) \ c_2Ebool_2ET))$

Definition 21 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap \ (c_2Ebool_2E_21 \ 2) \ (\lambda V2t \in 2.(c_2Ebool_2ET \ (t1 \ and \ t2)))) \ (\lambda V3t \in 2.(c_2Ebool_2ET \ (t1 \ or \ t2))))$

Definition 22 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 23 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 24 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 25 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 26 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap c_2Enum_2ESUC (ap$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Enumeral_2Eonecount : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eonecount \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (12)$$

Let $c_2Enumeral_2Eexactlog : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eexactlog \in (ty_2Enum_2Enum)^{ty_2Enum_2Enum} \quad (13)$$

Definition 27 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 28 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap (ap (ap (ap (c_2Ebool_2E$

Definition 29 We define $c_2Earithmetic_2EDIV2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap (ap c_2Earithmeti$

Let $c_2Enumeral_2Etexp_help : \iota$ be given. Assume the following.

$$c_2Enumeral_2Etexp_help \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (14)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (15)$$

Definition 30 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A_27a}). (\lambda V1x \in A_27$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (16)$$

Definition 31 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum. (ap (ap c_2Earithmeti$

Definition 32 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x.$

Definition 33 We define $c_2Enumeral_2Einternal_mult$ to be $c_2Earithmetic_2E_2A.$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m)) \quad (17)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (18)$$

Assume the following.

$$\begin{aligned} &(\forall V0n \in ty_2Enum_2Enum. (\forall V1k \in ty_2Enum_2Enum. (\\ &\forall V2r \in ty_2Enum_2Enum. ((\exists V3q \in ty_2Enum_2Enum. (\\ &(V1k = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A \\ &V3q) V0n)) V2r)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V2r) V0n)))) \Rightarrow (\\ &(ap (ap c_2Earithmetic_2EMOD V1k) V0n) = V2r))))))) \end{aligned} \quad (19)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\forall V1k \in ty_2Enum_2Enum. (\\ &(p (ap (ap c_2Eprim_rec_2E_3C V1k) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD \\ &V1k) V0n) = V1k)))) \quad (20)$$

Assume the following.

$$\begin{aligned} &(\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\ &c_2Enum_2E0) V0n)) \Rightarrow (((ap (ap c_2Earithmetic_2EDIV V0n) V0n) = \\ &(ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \wedge \\ &((ap (ap c_2Earithmetic_2EMOD V0n) V0n) = c_2Enum_2E0)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} &(\forall V0n \in ty_2Enum_2Enum. (\forall V1a \in ty_2Enum_2Enum. (\\ &(\neg((ap (ap (ap c_2Ebit_2EBITS V0n) V0n) V1a) = (ap c_2Earithmetic_2ENUMERAL \\ &(ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \Leftrightarrow ((ap (\\ &ap (ap c_2Ebit_2EBITS V0n) V0n) V1a) = c_2Enum_2E0)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1a \in ty_2Enum_2Enum. (\\
& \forall V2b \in ty_2Enum_2Enum. ((ap (ap (ap c_2Ebit_2EBITS (ap c_2Enum_2ESUC \\
& V0n)) (ap c_2Enum_2ESUC V0n)) (ap (ap c_2Earithmetic_2E_2B V1a) \\
& V2b)) = (ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Ebit_2EBITS (ap c_2Enum_2ESUC \\
& V0n)) (ap c_2Enum_2ESUC V0n)) V1a)) (ap (ap (ap c_2Ebit_2EBITS (\\
& ap c_2Enum_2ESUC V0n)) (ap c_2Enum_2ESUC V0n)) V2b))) (ap (ap (ap \\
& c_2Ebit_2EBITS (ap c_2Enum_2ESUC V0n)) (ap c_2Enum_2ESUC V0n)) \\
& (ap (ap c_2Earithmetic_2E_2B (ap (ap (ap c_2Ebit_2EBITS V0n) c_2Enum_2E0) \\
& V1a)) (ap (ap (ap c_2Ebit_2EBITS V0n) c_2Enum_2E0) V2b)))) (ap \\
& c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))))))) \\
& (23)
\end{aligned}$$

Assume the following.

$$True \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (29)
\end{aligned}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_27a. & nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\ V0t1) V1t2) = V1t2)))) \end{aligned} \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A_27a. & nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ V5y_27)))))))))) \end{aligned} \quad (35)$$

Assume the following.

$$(\forall V0v \in 2. ((p (ap c_2Ebool_2EBOUNDED V0v)) \Leftrightarrow True)) \quad (36)$$

Assume the following.

$$\begin{aligned} (((ap c_2Enum_2ESUC c_2Earithmetic_2EZERO) = (ap c_2Earithmetic_2EBIT1 \\ c_2Earithmetic_2EZERO)) \wedge ((\forall V0n \in ty_2Enum_2Enum. ((ap \\ c_2Enum_2ESUC (ap c_2Earithmetic_2EBIT1 V0n)) = (ap c_2Earithmetic_2EBIT2 \\ V0n))) \wedge (\forall V1n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC (ap c_2Earithmetic_2EBIT2 \\ V1n)) = (ap c_2Earithmetic_2EBIT1 (ap c_2Enum_2ESUC V1n))))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP \\
& c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP V14n) c_2Enum_2E0) = \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& (ap (ap c_2Earithmetic_2EEEXP V15n) V16m)))))) \wedge (((ap c_2Enum_2ESUC \\
& c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)))) \wedge ((\forall V17n \in ty_2Enum_2Enum. \\
& (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Enum_2ESUC V17n)))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO)))) \wedge \\
& ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL \\
& V32n)) (ap c_2Earithmetic_2ENUMERAL V32n))) \Leftrightarrow True))) \wedge ((\forall V33n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL \\
& V33n)) (ap c_2Earithmetic_2ENUMERAL V33n))) \Leftrightarrow True)))
\end{aligned}$$

Assume the following.

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
 & ((c_2Earithmetic_2EZERO = (ap c_2Earithmetic_2EBIT1 V0n)) \Leftrightarrow False) \wedge \\
 & (((ap c_2Earithmetic_2EBIT1 V0n) = c_2Earithmetic_2EZERO) \Leftrightarrow \\
 & False) \wedge (((c_2Earithmetic_2EZERO = (ap c_2Earithmetic_2EBIT2 \\
 & V0n)) \Leftrightarrow False) \wedge (((ap c_2Earithmetic_2EBIT2 V0n) = c_2Earithmetic_2EZERO) \Leftrightarrow \\
 & False) \wedge (((ap c_2Earithmetic_2EBIT1 V0n) = (ap c_2Earithmetic_2EBIT2 \\
 & V1m)) \Leftrightarrow False) \wedge (((ap c_2Earithmetic_2EBIT2 V0n) = (ap c_2Earithmetic_2EBIT1 \\
 & V1m)) \Leftrightarrow False) \wedge (((ap c_2Earithmetic_2EBIT1 V0n) = (ap c_2Earithmetic_2EBIT1 \\
 & V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap c_2Earithmetic_2EBIT2 V0n) = (ap c_2Earithmetic_2EBIT2 \\
 & V1m)) \Leftrightarrow (V0n = V1m))))))) \\
 \end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
 & ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 \\
 & V0n)) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
 & (ap c_2Earithmetic_2EBIT2 V0n)) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
 & V0n) c_2Earithmetic_2EZERO) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
 & (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT1 V1m)) \Leftrightarrow \\
 & (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m)) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
 & (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT2 V1m)) \Leftrightarrow \\
 & (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m)) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
 & (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT2 V1m)) \Leftrightarrow \\
 & (\neg(p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT1 V1m)) \Leftrightarrow \\
 & (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))) \\
 \end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. (((ap c_2Enumeral_2EiDUB (ap c_2Earithmetic_2EBIT1 \\
 & V0n)) = (ap c_2Earithmetic_2EBIT2 (ap c_2Enumeral_2EiDUB V0n))) \wedge \\
 & (((ap c_2Enumeral_2EiDUB (ap c_2Earithmetic_2EBIT2 V0n)) = (ap \\
 & c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 V0n))) \wedge ((ap \\
 & c_2Enumeral_2EiDUB c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO))) \\
 \end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. ((p (ap c_2Earithmetic_2EEVEN c_2Earithmetic_2EZERO)) \wedge \\
 & ((p (ap c_2Earithmetic_2EEVEN (ap c_2Earithmetic_2EBIT2 V0n)) \wedge \\
 & ((\neg(p (ap c_2Earithmetic_2EEVEN (ap c_2Earithmetic_2EBIT1 V0n))) \wedge \\
 & ((\neg(p (ap c_2Earithmetic_2EODD c_2Earithmetic_2EZERO)) \wedge ((\\
 & \neg(p (ap c_2Earithmetic_2EODD (ap c_2Earithmetic_2EBIT2 V0n))) \wedge \\
 & (p (ap c_2Earithmetic_2EODD (ap c_2Earithmetic_2EBIT1 V0n))))))) \\
 \end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
 & ((\forall V0x \in ty_2Enum_2Enum.((ap (ap c_2Enumeral_2Eonecount \\
 & c_2Earithmetic_2EZERO) V0x) = V0x)) \wedge (\forall V1n \in ty_2Enum_2Enum. \\
 & (\forall V2x \in ty_2Enum_2Enum.((ap (ap c_2Enumeral_2Eonecount \\
 & (ap c_2Earithmetic_2EBIT1 V1n)) V2x) = (ap (ap c_2Enumeral_2Eonecount \\
 & V1n) (ap c_2Enum_2ESUC V2x)))))) \wedge (\forall V3n \in ty_2Enum_2Enum. \\
 & (\forall V4x \in ty_2Enum_2Enum.((ap (ap c_2Enumeral_2Eonecount \\
 & (ap c_2Earithmetic_2EBIT2 V3n)) V4x) = c_2Earithmetic_2EZERO)))) \\
 &
 \end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
 & (((ap c_2Enumeral_2Eexactlog c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO) \wedge \\
 & ((\forall V0n \in ty_2Enum_2Enum.((ap c_2Enumeral_2Eexactlog \\
 & (ap c_2Earithmetic_2EBIT1 V0n)) = c_2Earithmetic_2EZERO)) \wedge (\forall V1n \in \\
 & ty_2Enum_2Enum.((ap c_2Enumeral_2Eexactlog (ap c_2Earithmetic_2EBIT2 \\
 & V1n)) = (ap (ap (c_2Ebool_2LET ty_2Enum_2Enum ty_2Enum_2Enum) \\
 & (\lambda V2x \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
 & (ap (ap (c_2Emin_2E_3D ty_2Enum_2Enum) V2x) c_2Earithmetic_2EZERO) \\
 & c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 V2x)))) (ap \\
 & (ap c_2Enumeral_2Eonecount V1n) c_2Earithmetic_2EZERO))))))) \\
 &
 \end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1x \in ty_2Enum_2Enum. (\forall V2y \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2A c_2Earithmetic_2EZERO) \\
V0n) = c_2Earithmetic_2EZERO) \wedge (((ap (ap c_2Earithmetic_2E_2A \\
V0n) c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO) \wedge (((ap \\
(ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2EBIT1 V1x)) (ap \\
c_2Earithmetic_2EBIT1 V2y)) = (ap (ap c_2Enumeral_2Einternal_mult \\
(ap c_2Earithmetic_2EBIT1 V1x)) (ap c_2Earithmetic_2EBIT1 V2y))) \wedge \\
(((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2EBIT1 V1x)) \\
(ap c_2Earithmetic_2EBIT2 V2y)) = (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum \\
ty_2Enum_2Enum) (\lambda V3n \in ty_2Enum_2Enum. (ap (ap (c_2Ebool_2ECOND \\
ty_2Enum_2Enum) (ap c_2Earithmetic_2EODD V3n)) (ap (ap c_2Enumeral_2Eexp_help \\
(ap c_2Earithmetic_2EDIV2 V3n)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 \\
V1x)))) (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmetic_2EBIT1 \\
V1x)) (ap c_2Earithmetic_2EBIT2 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
(ap c_2Earithmetic_2EBIT2 V2y)))) \wedge (((ap (ap c_2Earithmetic_2E_2A \\
(ap c_2Earithmetic_2EBIT2 V1x)) (ap c_2Earithmetic_2EBIT1 V2y)) = \\
(ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V4m \in \\
ty_2Enum_2Enum. (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
(ap c_2Earithmetic_2EODD V4m)) (ap (ap c_2Enumeral_2Eexp_help \\
(ap c_2Earithmetic_2EDIV2 V4m)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 \\
V2y)))))) (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmetic_2EBIT2 \\
V1x)) (ap c_2Earithmetic_2EBIT1 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
(ap c_2Earithmetic_2EBIT2 V1x)))) \wedge ((ap (ap c_2Earithmetic_2E_2A \\
(ap c_2Earithmetic_2EBIT2 V1x)) (ap c_2Earithmetic_2EBIT2 V2y)) = \\
(ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V5m \in \\
ty_2Enum_2Enum. (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) \\
(\lambda V6n \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
(ap c_2Earithmetic_2EODD V5m)) (ap (ap c_2Enumeral_2Eexp_help \\
(ap c_2Earithmetic_2EDIV2 V5m)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT2 \\
V2y)))))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap c_2Earithmetic_2EODD \\
V6n)) (ap (ap c_2Enumeral_2Eexp_help (ap c_2Earithmetic_2EDIV2 \\
V6n)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT2 V1x)))) \\
(ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmetic_2EBIT2 \\
V1x)) (ap c_2Earithmetic_2EBIT2 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
(ap c_2Earithmetic_2EBIT2 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
(ap c_2Earithmetic_2EBIT2 V1x))))))))))) \\
(46)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Enumeral_2Einternal_mult c_2Earithmetic_2EZERO) \\
& V0n) = c_2Earithmetic_2EZERO) \wedge (((ap (ap c_2Enumeral_2Einternal_mult \\
& V0n) c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO) \wedge (((ap \\
& (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmetic_2EBIT1 \\
& V0n)) V1m) = (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Enumeral_2EiDUB (ap (ap c_2Enumeral_2Einternal_mult \\
& V0n) V1m))) V1m))) \wedge ((ap (ap c_2Enumeral_2Einternal_mult (ap \\
& c_2Earithmetic_2EBIT2 V0n)) V1m) = (ap c_2Enumeral_2EiDUB (ap \\
& c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Enumeral_2Einternal_mult \\
& V0n) V1m)))))))))) \\
& \tag{47}
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1a \in ty_2Enum_2Enum. (\\
& \forall V2b \in ty_2Enum_2Enum. (p (ap (ap c_2Ebit_2EBIT (ap c_2Enum_2ESUC \\
& V0n)) (ap (ap c_2Earithmetic_2E_2B V1a) V2b))) \Leftrightarrow (p (ap (ap (ap (c_2Ebool_2ECOND \\
& 2) (ap (ap c_2Ebit_2EBIT (ap c_2Enum_2ESUC V0n)) (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap (ap c_2Ebit_2EBITS V0n) c_2Enum_2E0) V1a)) (ap (ap (ap c_2Ebit_2EBITS \\
& V0n) c_2Enum_2E0) V2b))) (ap (ap (c_2Emin_2E_3D 2) (ap (ap c_2Ebit_2EBIT \\
& (ap c_2Enum_2ESUC V0n)) V1a)) (ap (ap c_2Ebit_2EBIT (ap c_2Enum_2ESUC \\
& V0n)) V2b))) (ap c_2Ebool_2E_7E (ap (ap (c_2Emin_2E_3D 2) (ap (\\
& ap c_2Ebit_2EBIT (ap c_2Enum_2ESUC V0n)) V1a)) (ap (ap c_2Ebit_2EBIT \\
& (ap c_2Enum_2ESUC V0n)) V2b)))))))) \\
& \tag{47}
\end{aligned}$$