

thm_2Ebit_2EBITS__SUM2
(TMH8BcNNNoNvC75eHEtYGWHkzC46Yhp1cEsN)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{2}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{omega} \tag{5}$$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$.

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ P))\ a))$.

Definition 6 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$.

Definition 8 We define $c_Ebool_E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21) 2) (\lambda V2t \in 2.))$

Let $c_Enum_EREP_num : \iota$ be given. Assume the following.

$$c_Enum_EREP_num \in (\omega^{ty_Enum_Enum}) \quad (6)$$

Let $c_Enum_ESUC_REP : \iota$ be given. Assume the following.

$$c_Enum_ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Definition 9 We define c_Enum_ESUC to be $\lambda V0m \in ty_Enum_Enum.(ap c_Enum_EABS_num (ap (c_Enum_EABS_num) m))$

Definition 10 We define $c_Emin_E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \Rightarrow P x)) \text{ of type } \iota \Rightarrow \iota.$

Definition 11 We define $c_Ebool_E_3F$ to be $\lambda A.\lambda P : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_Emin_E_40) P)))$

Definition 12 We define $c_Eprim_rec_E_3C$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum.(ap (c_Eprim_rec) m n)$

Definition 13 We define $c_Earithmetic_EZERO$ to be c_Enum_E0 .

Definition 14 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_Enum_Enum.(ap (ap c_Earithmetic_E2) n)$

Definition 15 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Let $c_Earithmetic_EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_EEXP \in ((ty_Enum_Enum)^{ty_Enum_Enum})^{ty_Enum_Enum} \quad (8)$$

Let $c_Earithmetic_EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_EMOD \in ((ty_Enum_Enum)^{ty_Enum_Enum})^{ty_Enum_Enum} \quad (9)$$

Let $c_Earithmetic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_EDIV \in ((ty_Enum_Enum)^{ty_Enum_Enum})^{ty_Enum_Enum} \quad (10)$$

Definition 16 We define $c_Ebit_EDIV_EXP$ to be $\lambda V0x \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum.(ap (ap c_Earithmetic_EDIV) x n)$

Let $c_Earithmetic_E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2D \in ((ty_Enum_Enum)^{ty_Enum_Enum})^{ty_Enum_Enum} \quad (11)$$

Definition 17 We define $c_Ebit_EMOD_EXP$ to be $\lambda V0x \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum.(ap (ap c_Earithmetic_EMOD) x n)$

Definition 18 We define c_Ebit_EBITS to be $\lambda V0h \in ty_Enum_Enum.\lambda V1l \in ty_Enum_Enum.\lambda V2m \in ty_Enum_Enum.(ap (ap (ap c_Ebit_EBIT2) h) l) m)$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad c_2Enum_2E0) V0n)) \Rightarrow (\forall V1q \in ty_2Enum_2Enum. (\forall V2r \in \\
& \quad ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap (ap c_2Earithmetic_2E_2A V1q) V0n)) V2r)) V0n) = (ap (ap c_2Earithmetic_2EMOD \\
& \quad \quad V2r) V0n))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) \\
& \quad (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL (ap \\
& \quad \quad c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V0n))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty_2Enum_2Enum. (\forall V1l \in ty_2Enum_2Enum. (\\
& \quad \forall V2n \in ty_2Enum_2Enum. ((ap (ap (ap c_2Ebit_2EBITS V0h) V1l) \\
& \quad V2n) = (ap (ap c_2Earithmetic_2EDIV (ap (ap c_2Earithmetic_2EMOD \\
& \quad \quad V2n) (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL \\
& \quad \quad (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) (ap c_2Enum_2ESUC \\
& \quad \quad V0h)))) (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL \\
& \quad \quad (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V1l))))))
\end{aligned} \tag{14}$$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& \quad (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\
& \quad \quad p V0t))))))
\end{aligned} \tag{17}$$

Theorem 1

$$\begin{aligned}
& (\forall V0h \in ty_2Enum_2Enum. (\forall V1l \in ty_2Enum_2Enum. (\\
& \quad \forall V2a \in ty_2Enum_2Enum. (\forall V3b \in ty_2Enum_2Enum. ((\\
& \quad \quad ap (ap (ap c_2Ebit_2EBITS V0h) V1l) (ap (ap c_2Earithmetic_2E_2B \\
& \quad \quad (ap (ap c_2Earithmetic_2E_2A V2a) (ap (ap c_2Earithmetic_2EEXP \\
& \quad \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& \quad \quad (ap c_2Enum_2ESUC V0h)))))) V3b)) = (ap (ap (ap c_2Ebit_2EBITS V0h) \\
& \quad \quad V1l) V3b))))))
\end{aligned}$$