

# thm\_2Ebit\_2EBITS\_\_THM2

(TMG6rtQU6WwzKSN7RsQc7eLyjPNkNdQuxmQ)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota))$

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c\_2Emin\_2E\_40 A) (c\_2Emin\_2E\_3D A))))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (2)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (3)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (4)$$

**Definition 4** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (6)$$

**Definition 5** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)))$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ P)))$

**Definition 7** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

**Definition 8** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2A\ n))$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_21))$

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

**Definition 13** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (c\_2Eprim\_rec\_2E\_3C\ m)\ n)$

**Definition 14** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

**Definition 15** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (c\_2Earithmetic\_2E\_3C\_3D\ m)\ n)$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 16** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 17** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2A\ n)\ n)$

**Definition 18** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2EEEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 19** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (c\_2Ebit\_2EDIV\_2EXP\ x)\ n)$

**Definition 20** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 21** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2Enum\_2Enum. \lambda V2n \in ty\_2Enum\_2Enum.$

Assume the following.

$$((\forall V0m \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2EEEXP V0m) c\_2Enum\_2E0) = (ap (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge (\forall V1m \in ty\_2Enum\_2Enum. (\forall V2n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2EEEXP V1m) (ap c\_2Enum\_2ESUC V2n)) = (ap (ap c\_2Earithmetic\_2E\_2A V1m) (ap (ap c\_2Earithmetic\_2EEEXP V1m) V2n))))))) \quad (12)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B V1n) V0m)))) \quad (13)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B V1n) V0m)))) \quad (14)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C V1n) V0m)) \Rightarrow (\exists V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V2p) V1n) = V0m)))))) \quad (15)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1n) V0m))))))) \quad (16)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2D V0m) V1n) = c\_2Enum\_2E0) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)))))) \quad (17)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V2p))))))) \quad (18)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in ty\_2Enum\_2Enum. (\forall V1q \in ty\_2Enum\_2Enum. (\forall V2n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2EEXP V2n) \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0p) V1q)) = (ap (ap c\_2Earithmetic\_2E\_2A \\
 & (ap (ap c\_2Earithmetic\_2EEXP V2n) V0p)) (ap (ap c\_2Earithmetic\_2EEXP \\
 & V2n) V1q)))))))
 \end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in ty\_2Enum\_2Enum. (\forall V1c \in ty\_2Enum\_2Enum. (\forall V2D \in ty\_2Enum\_2Enum. \\
 & (ap (ap c\_2Earithmetic\_2E\_2D (ap (ap c\_2Earithmetic\_2E\_2B V0a) \\
 & V1c)) V1c) = V0a)))
 \end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0k \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2EMOD V0k) \\
 & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = \\
 & c\_2Enum\_2E0)))
 \end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0r \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2C \in ty\_2Enum\_2Enum. \\
 & (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0r) V1n)) \Rightarrow ((ap (ap c\_2Earithmetic\_2EDIV \\
 & V0r) V1n) = c\_2Enum\_2E0))))
 \end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2k \in ty\_2Enum\_2Enum. \\
 & ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V1n)) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
 & c\_2Enum\_2E0) V2k))) \Rightarrow ((ap (ap c\_2Earithmetic\_2EMOD (ap (ap c\_2Earithmetic\_2EDIV \\
 & V0m) V1n)) V2k) = (ap (ap c\_2Earithmetic\_2EDIV (ap (ap c\_2Earithmetic\_2EMOD \\
 & V0m) (ap (ap c\_2Earithmetic\_2E\_2A V1n) V2k))) V1n))))))
 \end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) \\
 & (ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap \\
 & c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V0n))))
 \end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. (\forall V2C \in ty\_2Enum\_2Enum. \\
 & (p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap (ap c\_2Earithmetic\_2EMOD V1k) \\
 & (ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap \\
 & c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V0n))) (ap \\
 & ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\
 & c\_2Earithmetic\_2EZERO))) V0n))))))
 \end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in ty\_2Enum\_2Enum. (\forall V1b \in ty\_2Enum\_2Enum. ( \\
 & (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0a) V1b)) \Rightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & (ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap \\
 & c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V0a)) (ap (ap \\
 & c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\
 & c\_2Earithmetic\_2EZERO))) V1b))))))) \\
 \end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. ( \\
 & \forall V2n \in ty\_2Enum\_2Enum. ((ap (ap (ap c\_2Ebit\_2EBITS V0h) V1l) \\
 & V2n) = (ap (ap c\_2Earithmetic\_2EMOD (ap (ap c\_2Earithmetic\_2EDIV \\
 & V2n) (ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V1l))) ( \\
 & ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap \\
 & c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) (ap (ap c\_2Earithmetic\_2E\_2D \\
 & (ap c\_2Enum\_2ESUC V0h)) V1l))))))) \\
 \end{aligned} \tag{27}$$

Assume the following.

$$True \tag{28}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{29}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
 & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
 & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \\
 \end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
 & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
 & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \\
 \end{aligned} \tag{31}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{32}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (34)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (35)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27))))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \quad (36)$$

### Theorem 1

$$\begin{aligned} & (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. ( \\ & \quad \forall V2n \in ty\_2Enum\_2Enum. ((ap (ap (ap c_2Ebit_2EBITS V0h) V1l) \\ & \quad V2n) = (ap (ap c_2Earithmetic_2EDIV (ap (ap c_2Earithmetic_2EMOD \\ & \quad V2n) (ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL \\ & \quad (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) (ap c_2Enum_2ESUC \\ & \quad V0h))) (ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL \\ & \quad (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V1l))))))) \end{aligned}$$