

# thm\_2Ebit\_2EBITWISE\_\_COR (TMYLLRL-Litz8PCM2yNmEfLepvJKCKdhHszF)

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Let  $c\_2Enum\_2ZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ZERO\_REP \in \omega \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Enum\_2ABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2ABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (3)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2ABS\_num\ c\_2Enum\_2ZERO\_REP)$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2SUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2SUC\_REP \in (\omega^{\omega}) \quad (5)$$

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. \lambda P \in (2^{A-27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V0P \in 2.V0P)))$

**Definition 5** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2ABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 6** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2D n) 0)$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 7** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 8** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2D n) 1)$

**Definition 9** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2EEEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 10** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2Earithmetic\_2EDIV (c\_2Earithmetic\_2EEEXP n) x)$

**Definition 11** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2Earithmetic\_2EMOD (c\_2Earithmetic\_2EEEXP n) x)$

**Definition 12** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V2t \in 2.(c\_2Earithmetic\_2EDIV h l) t$

Let  $c\_2Ebit\_2EBITWISE : \iota$  be given. Assume the following.

$$c\_2Ebit\_2EBITWISE \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{((2^2)^2)})^{ty\_2Enum\_2Enum} \quad (11)$$

**Definition 13** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2Earithmetic\_2EDIV (c\_2Earithmetic\_2EEEXP n) b)$

**Definition 14** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 15** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 16** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21 2))$

**Definition 17** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.(c\_2Earithmetic\_2EDIV t1 t2) t)))$

**Definition 18** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge P x)) \text{ else } (\lambda x.x \in A \wedge \neg P x)$  of type  $\iota \Rightarrow \iota$ .

**Definition 19** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 20** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum. (((ap\ (ap\ c\_2Earithmetic\_2EEEXP \\ & (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) \\ & V0n) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\ & c\_2Earithmetic\_2EZERO))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2EEEXP\ V0n) \\ & (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) = \\ & V0n))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ ( \\ & ap\ c\_2Enum\_2ESUC\ V0a))\ V0a) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap \\ & c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. ( \\ & \forall V2n \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Ebit\_2EBITS\ V0h)\ V1l) \\ & V2n) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ (ap\ (ap\ c\_2Earithmetic\_2EDIV \\ & V2n)\ (ap\ (ap\ c\_2Earithmetic\_2EEEXP\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))\ V1l)))\ ( \\ & ap\ (ap\ c\_2Earithmetic\_2EEEXP\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap \\ & c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D \\ & (ap\ c\_2Enum\_2ESUC\ V0h))\ V1l))))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \forall V2op \in ((2^2)^2). (\forall V3a \in ty\_2Enum\_2Enum. (\forall V4b \in \\ & ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0x)\ V1n)) \Rightarrow (( \\ & p\ (ap\ (ap\ c\_2Ebit\_2EBIT\ V0x))\ (ap\ (ap\ (ap\ c\_2Ebit\_2EBITWISE\ V1n) \\ & V2op)\ V3a)\ V4b))) \Leftrightarrow (p\ (ap\ (ap\ V2op\ (ap\ (ap\ c\_2Ebit\_2EBIT\ V0x)\ V3a)) \\ & (ap\ (ap\ c\_2Ebit\_2EBIT\ V0x))\ V4b))))))))))) \end{aligned} \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \end{aligned} \quad (17)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0x \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V4b \in \\ & \quad \forall V2op \in ((2^2)^2).(\forall V3a \in ty\_2Enum\_2Enum.(\forall V4b \in \\ & \quad \quad ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C V0x) V1n)) \Rightarrow (( \\ & \quad \quad \quad p (ap (ap V2op (ap (ap c\_2Ebit\_2EBIT V0x) V3a)) (ap (ap c\_2Ebit\_2EBIT \\ & \quad \quad \quad V0x) V4b))) \Rightarrow ((ap (ap c\_2Earithmetic\_2EMOD (ap (ap c\_2Earithmetic\_2EDIV \\ & \quad \quad (ap (ap (ap (ap c\_2Ebit\_2EBITWISE V1n) V2op) V3a) V4b)) (ap (ap c\_2Earithmetic\_2EEXP \\ & \quad \quad \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\ & \quad \quad \quad V0x))) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\ & \quad \quad \quad c\_2Earithmetic\_2EZERO))) = (ap c\_2Earithmetic\_2ENUMERAL (ap \\ & \quad \quad \quad c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))))))) \end{aligned}$$