

thm\_2Ebit\_2EBITWISE\_\_ONE\_\_COMP\_\_LEM  
 (TMZRQN-  
 qEf44HqMvgUTQiKXRHKEwT12SyncS)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1P \in 2.V1P))\ (\lambda V2P \in 2.V2P))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (c\_2Earithmic\_2EZERO\ c\_2Enum\_2E0\ c\_2Enum\_2EREP\_num\ c\_2Enum\_2ESUC\_REP))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (6)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B))$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (7)$$

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V0t2)))$

**Definition 12** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then  $(the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.V0t2)))$

**Definition 14** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2E\_21 2))))$

Let  $c\_2Ebit\_2EBITWISE : \iota$  be given. Assume the following.

$$c\_2Ebit\_2EBITWISE \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{((2^2)^2)})^{ty\_2Enum\_2Enum} \quad (8)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (9)$$

**Definition 15** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V0x$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (10)$$

**Definition 16** We define  $c\_2Ebit\_2ESLICE$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.V0h$

**Definition 17** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B))$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (11)$$

**Definition 18** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 19** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 20** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

**Definition 21** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E$

**Definition 22** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 23** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 24** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 25** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 26** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 27** We define  $c\_2Enumeral\_2EiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2ESUC (ap$

**Definition 28** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Enumeral\_2EiSUB : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2EiSUB \in (((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^2 \quad (12)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (13)$$

Let  $c\_2Enumeral\_2Eonecount : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2Eonecount \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (14)$$

Let  $c\_2Enumeral\_2Eexactlog : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2Eexactlog \in (ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum} \quad (15)$$

**Definition 29** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2E$

**Definition 30** We define  $c\_2Earithmetic\_2EDIV2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

Let  $c\_2Enumeral\_2Eteexp\_help : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2Eteexp\_help \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (16)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (17)$$

**Definition 31** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})ty\_2Enum\_2Enum) \quad (18)$$

**Definition 32** We define  $c\_2Enumeral\_2EiDUB$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2A V0x))$

**Definition 33** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 34** We define  $c\_2Enumeral\_2Einternal\_mult$  to be  $c\_2Earithmetic\_2E\_2A$ .

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\ V0m) c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\ & c\_2Earithmetic\_2EZERO)))) \wedge (\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in \\ & ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V1m) (ap c\_2Enum\_2ESUC \\ & V2n)) = (ap (ap c\_2Earithmetic\_2E\_2A V1m) (ap (ap c\_2Earithmetic\_2EEXP \\ & V1m) V2n))))))) \end{aligned} \quad (19)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m)) \quad (20)$$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge ((ap ( \\ & ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B \\ & (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\ & V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\ & V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n))))))) \end{aligned} \quad (21)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\ V1n) V0m)))) \quad (22)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\ V1n) V0m)))) \quad (23)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
& (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0n)) (ap \\
& c\_2Enum\_2ESUC V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) \\
& \quad V1m))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad (ap c\_2Enum\_2ESUC V0m)) V1n))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V0n)))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V1n) V0m))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2E\_2D \\
& c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge ((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V0m) c\_2Enum\_2E0) = V0m)))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \\
& \quad V0m) = V0m))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
& (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2A \\
& V1n) V0m))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2A (ap \\
& (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap (ap c\_2Earithmetic\_2E\_2A V0m) V2p)) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V1n) V2p))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2A V2p) \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap (ap c\_2Earithmetic\_2E\_2A V2p) V0m)) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V2p) V1n))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2A V0m) \\
& (ap (ap c\_2Earithmetic\_2E\_2A V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2A \\
& (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) V2p))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
& \quad \quad ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p))))))
\end{aligned} \tag{36}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V0m))) \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Enum\_2Enum. (\forall V1b \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2c \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2D V0a) \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2B V1b) V2c)) = (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad \quad (ap (ap c\_2Earithmetic\_2E\_2D V0a) V1b)) V2c))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\
& \quad V0n) = (ap (ap c\_2Earithmetic\_2E\_2B V0n) V0n)))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Enum\_2Enum. (\forall V1c \in ty\_2Enum\_2Enum. ( \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2D (ap (ap c\_2Earithmetic\_2E\_2B V0a) \\
& \quad \quad V1c)) V1c) = V0a)))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p ( \\
& \quad \quad ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad \quad V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (\neg (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad \quad (ap c\_2Enum\_2ESUC V1n)) V0m))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& V1n)) V0m))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap \\
& c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))) V0n)))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B (ap \\
& (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p) = (ap (ap (ap (c\_2Ebool\_2ECOND \\
& ty\_2Enum\_2Enum) (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) V2p) \\
& (ap (ap c\_2Earithmetic\_2E\_2D (ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
& V2p)) V1n))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2D (ap \\
& (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p) = (ap (ap c\_2Earithmetic\_2E\_2D \\
& V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. ((V0m = (ap (ap c\_2Earithmetic\_2E\_2D \\
& V1n) V2p)) \Leftrightarrow (((ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p) = V1n) \vee ((p \\
& (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) c\_2Enum\_2E0)) \wedge (p (ap (ap \\
& c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (\forall V1a \in ty\_2Enum\_2Enum. \\
& (\forall V2b \in ty\_2Enum\_2Enum. ((p (ap V0P (ap (ap c\_2Earithmetic\_2E\_2D \\
& V1a) V2b))) \Leftrightarrow (\forall V3d \in ty\_2Enum\_2Enum. (((V2b = (ap (ap c\_2Earithmetic\_2E\_2B \\
& V1a) V3d)) \Rightarrow (p (ap V0P c\_2Enum\_2E0))) \wedge ((V1a = (ap (ap c\_2Earithmetic\_2E\_2B \\
& V2b) V3d)) \Rightarrow (p (ap V0P V3d))))))
\end{aligned} \tag{49}$$



Assume the following.

$$\begin{aligned}
& ((\forall V0op \in ((2^2)^2).(\forall V1x \in ty\_2Enum\_2Enum.(\forall V2y \in \\
& ty\_2Enum\_2Enum.((ap (ap (ap (ap c\_2Ebit\_2EBITWISE c\_2Enum\_2E0) \\
& V0op) V1x) V2y) = c\_2Enum\_2E0)))) \wedge (\forall V3n \in ty\_2Enum\_2Enum. \\
& (\forall V4op \in ((2^2)^2).(\forall V5x \in ty\_2Enum\_2Enum.(\forall V6y \in \\
& ty\_2Enum\_2Enum.((ap (ap (ap (ap c\_2Ebit\_2EBITWISE (ap c\_2Enum\_2ESUC \\
& V3n)) V4op) V5x) V6y) = (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap (ap ( \\
& ap c\_2Ebit\_2EBITWISE V3n) V4op) V5x) V6y)) (ap (ap c\_2Ebit\_2ESBIT \\
& (ap (ap V4op (ap (ap c\_2Ebit\_2EBIT V3n) V5x)) (ap (ap c\_2Ebit\_2EBIT \\
& V3n) V6y)))) V3n)))))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty\_2Enum\_2Enum.(\forall V1l \in ty\_2Enum\_2Enum.( \\
& \forall V2n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap \\
& (ap (ap c\_2Ebit\_2EBITS V0h) V1l) V2n)) (ap (ap c\_2Earithmetic\_2EEXP \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))) \\
& (ap (ap c\_2Earithmetic\_2E\_2D (ap c\_2Enum\_2ESUC V0h)) V1l))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1h \in ty\_2Enum\_2Enum.( \\
& \forall V2l \in ty\_2Enum\_2Enum.((ap (ap (ap c\_2Ebit\_2ESLICE V1h) \\
& V2l) V0n) = (ap (ap c\_2Earithmetic\_2E\_2A (ap (ap (ap c\_2Ebit\_2EBITS \\
& V1h) V2l) V0n)) (ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))) V2l))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1h \in ty\_2Enum\_2Enum.( \\
& (ap (ap (ap c\_2Ebit\_2ESLICE V1h) c\_2Enum\_2E0) V0n) = (ap (ap (ap c\_2Ebit\_2EBITS \\
& V1h) c\_2Enum\_2E0) V0n))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.( \\
& \forall V2l \in ty\_2Enum\_2Enum.(\forall V3n \in ty\_2Enum\_2Enum.(( \\
& (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1m)) V0h)) \wedge \\
& (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V2l) V1m))) \Rightarrow ((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap (ap (ap c\_2Ebit\_2ESLICE V0h) (ap c\_2Enum\_2ESUC V1m)) V3n)) ( \\
& ap (ap (ap c\_2Ebit\_2ESLICE V1m) V2l) V3n)) = (ap (ap (ap c\_2Ebit\_2ESLICE \\
& V0h) V2l) V3n))))))
\end{aligned} \tag{54}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1a \in ty\_2Enum\_2Enum. (\neg(p (ap (ap c\_2Ebit\_2EBIT V0n) V1a)) \Leftrightarrow ((ap (ap (ap c\_2Ebit\_2EBITS V0n) V0n) V1a) = c\_2Enum\_2E0)))) \quad (55)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1a \in ty\_2Enum\_2Enum. (\neg((ap (ap (ap c\_2Ebit\_2EBITS V0n) V0n) V1a) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \Leftrightarrow ((ap (ap (ap c\_2Ebit\_2EBITS V0n) V0n) V1a) = c\_2Enum\_2E0)))) \quad (56)$$

Assume the following.

$$True \quad (57)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (58)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (59)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \quad (60)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (61)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))) \quad (62)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (63)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))) \quad (64)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (65)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (66)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (67)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (68)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (69)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) V0t1) V1t2) = V1t2)))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (71)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge \neg(p V1B)))))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (73)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (74)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (75)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0b \in 2.(\forall V1f \in (A\_27b^{A\_27a}).(\forall V2g \in (A\_27b^{A\_27a}). \\ & \quad (\forall V3x \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (A\_27b^{A\_27a})) \\ & \quad V0b)\ V1f)\ V2g)\ V3x) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27b)\ V0b)\ (ap \\ & \quad V1f\ V3x))\ (ap\ V2g\ V3x))))))))) \end{aligned} \quad (76)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0f \in (A\_27b^{A\_27a}).(\forall V1b \in 2.(\forall V2x \in A\_27a. \\ & \quad (\forall V3y \in A\_27a.((ap\ V0f\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ & \quad V1b)\ V2x)\ V3y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27b)\ V1b)\ (ap\ V0f \\ & \quad V2x))\ (ap\ V0f\ V3y))))))))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ & \quad 2.(((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))))) \Rightarrow \\ & \quad (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & \quad (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\ & \quad (\forall V5y\_27 \in A\_27a.(((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & \quad ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ & \quad V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\ & \quad V5y\_27))))))))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ c\_2Enum\_2E0)) \wedge \\ & \quad (\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c\_2Enum\_2ESUC \\ & \quad V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (80)$$

Assume the following.

$$\begin{aligned} & (((ap\ c\_2Enum\_2ESUC\ c\_2Earithmetic\_2EZERO) = (ap\ c\_2Earithmetic\_2EBIT1 \\ & \quad c\_2Earithmetic\_2EZERO)) \wedge ((\forall V0n \in ty\_2Enum\_2Enum.((ap \\ & \quad c\_2Enum\_2ESUC\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2 \\ & \quad V0n))) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((ap\ c\_2Enum\_2ESUC\ (ap\ c\_2Earithmetic\_2EBIT2 \\ & \quad V1n)) = (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enum\_2ESUC\ V1n)))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ ( \\
& ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ ( \\
& ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ ( \\
& ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap \\
& c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC \\
& (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ c\_2Earithmetic\_2EZERO)) = ( \\
& ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ ( \\
& ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m))))))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) (ap c\_2Earithmic\_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) \\
& (ap c\_2Earithmic\_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0n) c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT1 V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT2 V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT1 V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1m) V0n))) \wedge ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT2 V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))))))))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D c\_2Earithmic\_2EZERO) V0n))) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT1 \\
& V0n)) c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& (ap c\_2Earithmic\_2EBIT2 V0n)) c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& (((ap c\_2Eprim\_rec\_2EPRE c\_2Earithmic\_2EZERO) = c\_2Earithmic\_2EZERO) \wedge \\
& (((ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT1 c\_2Earithmic\_2EZERO)) = \\
& c\_2Earithmic\_2EZERO) \wedge ((\forall V0n \in ty\_2Enum\_2Enum. ((ap \\
& c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT1 (ap c\_2Earithmic\_2EBIT1 \\
& V0n))) = (ap c\_2Earithmic\_2EBIT2 (ap c\_2Eprim\_rec\_2EPRE (ap \\
& c\_2Earithmic\_2EBIT1 V0n)))))) \wedge ((\forall V1n \in ty\_2Enum\_2Enum. \\
& ((ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT1 (ap c\_2Earithmic\_2EBIT2 \\
& V1n))) = (ap c\_2Earithmic\_2EBIT2 (ap c\_2Earithmic\_2EBIT1 \\
& V1n)))))) \wedge ((\forall V2n \in ty\_2Enum\_2Enum. ((ap c\_2Eprim\_rec\_2EPRE \\
& (ap c\_2Earithmic\_2EBIT2 V2n)) = (ap c\_2Earithmic\_2EBIT1 V2n))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Enum\_2Enum. (\forall V1b \in 2. (\forall V2n \in ty\_2Enum\_2Enum. \\
& (\forall V3m \in ty\_2Enum\_2Enum. (((ap (ap (ap c\_2Enumeral\_2EiSUB \\
& V1b) c\_2Earithmetic\_2EZERO) V0x) = c\_2Earithmetic\_2EZERO) \wedge ( \\
& ((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) c\_2Earithmetic\_2EZERO) = \\
V2n) \wedge (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT1 \\
V2n)) c\_2Earithmetic\_2EZERO) = (ap c\_2Enumeral\_2EiDUB V2n)) \wedge \\
& (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT1 \\
V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT1 \\
V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT1 \\
V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT1 \\
V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT2 \\
V2n)) c\_2Earithmetic\_2EZERO) = (ap c\_2Earithmetic\_2EBIT1 V2n)) \wedge \\
& (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT2 \\
V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT2 \\
V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT2 \\
V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge ((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT2 \\
V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))))))))))))))))) \\
& \hspace{15em} (87)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D V0n) \\
V1m)) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V1m) V0n)) (ap c\_2Earithmetic\_2ENUMERAL (ap (ap (ap c\_2Enumeral\_2EiSUB \\
& c\_2Ebool\_2ET) V0n) V1m))) c\_2Enum\_2E0)))) \\
& \hspace{15em} (88)
\end{aligned}$$



Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n)) = (ap \\
& \quad \quad c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))) \wedge ((ap \\
& \quad \quad c\_2Enumeral\_2EiDUB\ c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO)))) \\
& \hspace{15em} (89)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p\ (ap\ c\_2Earithmetic\_2EVEN\ c\_2Earithmetic\_2EZERO)) \wedge \\
& \quad ((p\ (ap\ c\_2Earithmetic\_2EVEN\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n))) \wedge \\
& \quad ((\neg(p\ (ap\ c\_2Earithmetic\_2EVEN\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)))) \wedge \\
& \quad \quad ((\neg(p\ (ap\ c\_2Earithmetic\_2EODD\ c\_2Earithmetic\_2EZERO))) \wedge (( \\
& \quad \quad \neg(p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n)))) \wedge \\
& \quad \quad (p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)))))))))) \\
& \hspace{15em} (90)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0acc \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& \quad (((ap\ (ap\ c\_2Enumeral\_2Etex\_help\ c\_2Earithmetic\_2EZERO)\ V0acc) = \\
& \quad (ap\ c\_2Earithmetic\_2EBIT2\ V0acc)) \wedge (((ap\ (ap\ c\_2Enumeral\_2Etex\_help \\
& \quad (ap\ c\_2Earithmetic\_2EBIT1\ V1n))\ V0acc) = (ap\ (ap\ c\_2Enumeral\_2Etex\_help \\
& \quad \quad (ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmetic\_2EBIT1\ V1n)))\ (ap \\
& \quad \quad c\_2Earithmetic\_2EBIT1\ V0acc))) \wedge ((ap\ (ap\ c\_2Enumeral\_2Etex\_help \\
& \quad (ap\ c\_2Earithmetic\_2EBIT2\ V1n))\ V0acc) = (ap\ (ap\ c\_2Enumeral\_2Etex\_help \\
& \quad \quad (ap\ c\_2Earithmetic\_2EBIT1\ V1n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V0acc)))))) \\
& \hspace{15em} (91)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (((ap\ (ap\ c\_2Earithmetic\_2EEXP \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO))) \\
& \quad c\_2Enum\_2E0) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad \quad c\_2Earithmetic\_2EZERO))) \wedge (((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap \\
& \quad c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO))) \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))) = \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ (ap\ c\_2Enumeral\_2Etex\_help \\
& \quad (ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)))\ c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ c\_2Earithmetic\_2ENUMERAL\ ( \\
& \quad ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad \quad (ap\ c\_2Earithmetic\_2EBIT2\ V0n))) = (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad \quad (ap\ (ap\ c\_2Enumeral\_2Etex\_help\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) \\
& \quad \quad c\_2Earithmetic\_2EZERO)))))) \\
& \hspace{15em} (92)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Enum\_2Enum.((ap (ap c\_2Enumeral\_2Eonecount \\
& c\_2Earithmetic\_2EZERO) V0x) = V0x)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum. \\
& (\forall V2x \in ty\_2Enum\_2Enum.((ap (ap c\_2Enumeral\_2Eonecount \\
& (ap c\_2Earithmetic\_2EBIT1 V1n)) V2x) = (ap (ap c\_2Enumeral\_2Eonecount \\
& V1n) (ap c\_2Enum\_2ESUC V2x)))))) \wedge ((\forall V3n \in ty\_2Enum\_2Enum. \\
& (\forall V4x \in ty\_2Enum\_2Enum.((ap (ap c\_2Enumeral\_2Eonecount \\
& (ap c\_2Earithmetic\_2EBIT2 V3n)) V4x) = c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{93}$$

Assume the following.

$$\begin{aligned}
& (((ap c\_2Enumeral\_2Eexactlog c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO) \wedge \\
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap c\_2Enumeral\_2Eexactlog ( \\
& ap c\_2Earithmetic\_2EBIT1 V0n)) = c\_2Earithmetic\_2EZERO)) \wedge ((\forall V1n \in \\
& ty\_2Enum\_2Enum.((ap c\_2Enumeral\_2Eexactlog (ap c\_2Earithmetic\_2EBIT2 \\
& V1n)) = (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) \\
& (\lambda V2x \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) \\
& (ap (ap (c\_2Emin\_2E\_3D ty\_2Enum\_2Enum) V2x) c\_2Earithmetic\_2EZERO)) \\
& c\_2Earithmetic\_2EZERO) (ap c\_2Earithmetic\_2EBIT1 V2x)))) (ap \\
& (ap c\_2Enumeral\_2Eonecount V1n) c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{94}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1x \in ty\_2Enum\_2Enum. ( \\
& \forall V2y \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmic\_2E\_2A c\_2Earithmic\_2EZERO) \\
& V0n) = c\_2Earithmic\_2EZERO) \wedge (((ap (ap c\_2Earithmic\_2E\_2A \\
& V0n) c\_2Earithmic\_2EZERO) = c\_2Earithmic\_2EZERO) \wedge ((ap \\
& (ap c\_2Earithmic\_2E\_2A (ap c\_2Earithmic\_2EBIT1 V1x)) (ap \\
& c\_2Earithmic\_2EBIT1 V2y)) = (ap (ap c\_2Enumeral\_2Einternal\_mult \\
& (ap c\_2Earithmic\_2EBIT1 V1x)) (ap c\_2Earithmic\_2EBIT1 V2y)))) \wedge \\
& (((ap (ap c\_2Earithmic\_2E\_2A (ap c\_2Earithmic\_2EBIT1 V1x)) \\
& (ap c\_2Earithmic\_2EBIT2 V2y)) = (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum \\
& ty\_2Enum\_2Enum) (\lambda V3n \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2ECOND \\
& ty\_2Enum\_2Enum) (ap c\_2Earithmic\_2EODD V3n)) (ap (ap c\_2Enumeral\_2Eexp\_help \\
& (ap c\_2Earithmic\_2EDIV2 V3n)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT1 \\
& V1x)))) (ap (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmic\_2EBIT1 \\
& V1x)) (ap c\_2Earithmic\_2EBIT2 V2y)))))) (ap c\_2Enumeral\_2Eexactlog \\
& (ap c\_2Earithmic\_2EBIT2 V2y)))) \wedge (((ap (ap c\_2Earithmic\_2E\_2A \\
& (ap c\_2Earithmic\_2EBIT2 V1x)) (ap c\_2Earithmic\_2EBIT1 V2y)) = \\
& (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) (\lambda V4m \in \\
& ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) \\
& (ap c\_2Earithmic\_2EODD V4m)) (ap (ap c\_2Enumeral\_2Eexp\_help \\
& (ap c\_2Earithmic\_2EDIV2 V4m)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT1 \\
& V2y)))) (ap (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmic\_2EBIT2 \\
& V1x)) (ap c\_2Earithmic\_2EBIT1 V2y)))))) (ap c\_2Enumeral\_2Eexactlog \\
& (ap c\_2Earithmic\_2EBIT2 V1x)))) \wedge ((ap (ap c\_2Earithmic\_2E\_2A \\
& (ap c\_2Earithmic\_2EBIT2 V1x)) (ap c\_2Earithmic\_2EBIT2 V2y)) = \\
& (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) (\lambda V5m \in \\
& ty\_2Enum\_2Enum. (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) \\
& (\lambda V6n \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) \\
& (ap c\_2Earithmic\_2EODD V5m)) (ap (ap c\_2Enumeral\_2Eexp\_help \\
& (ap c\_2Earithmic\_2EDIV2 V5m)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT2 \\
& V2y)))) (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) (ap c\_2Earithmic\_2EODD \\
& V6n)) (ap (ap c\_2Enumeral\_2Eexp\_help (ap c\_2Earithmic\_2EDIV2 \\
& V6n)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT2 V1x)))) \\
& (ap (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmic\_2EBIT2 \\
& V1x)) (ap c\_2Earithmic\_2EBIT2 V2y)))))) (ap c\_2Enumeral\_2Eexactlog \\
& (ap c\_2Earithmic\_2EBIT2 V2y)))) (ap c\_2Enumeral\_2Eexactlog \\
& (ap c\_2Earithmic\_2EBIT2 V1x))))))))) (95)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Enumeral\_2Einternal\_mult c\_2Earithmetic\_2EZERO) \\
V0n) = c\_2Earithmetic\_2EZERO) \wedge ((ap (ap c\_2Enumeral\_2Einternal\_mult \\
& V0n) c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO) \wedge ((ap \\
& (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmetic\_2EBIT1 \\
V0n)) V1m) = (ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Enumeral\_2EiDUB (ap (ap c\_2Enumeral\_2Einternal\_mult \\
& V0n) V1m))) V1m))) \wedge ((ap (ap c\_2Enumeral\_2Einternal\_mult (ap \\
& c\_2Earithmetic\_2EBIT2 V0n)) V1m) = (ap c\_2Enumeral\_2EiDUB (ap \\
& c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Enumeral\_2Einternal\_mult \\
& V0n) V1m))) V1m)))))))))
\end{aligned} \tag{96}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{97}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{98}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{100}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \tag{101}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))
\end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge ((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q)) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q) \vee \neg(p \ V0p))))))
\end{aligned} \tag{105}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0n \in \text{ty\_2Enum\_2Enum}. (\forall V1a \in \text{ty\_2Enum\_2Enum}. ( \\
& \forall V2b \in \text{ty\_2Enum\_2Enum}. ((\text{ap} (\text{ap} (\text{ap} (\text{ap} \ \text{c\_2Ebit\_2EBITWISE} \\
& (\text{ap} \ \text{c\_2Enum\_2ESUC} \ V0n)) (\lambda V3x \in 2. (\lambda V4y \in 2. (\text{ap} \ \text{c\_2Ebool\_2E} \\
& \text{V3x})))) V1a) \ V2b) = (\text{ap} (\text{ap} \ \text{c\_2Earithmetic\_2E\_2D} (\text{ap} (\text{ap} \ \text{c\_2Earithmetic\_2E\_2D} \\
& (\text{ap} (\text{ap} \ \text{c\_2Earithmetic\_2EEXP} (\text{ap} \ \text{c\_2Earithmetic\_2ENUMERAL} (\text{ap} \\
& \text{c\_2Earithmetic\_2EBIT2} \ \text{c\_2Earithmetic\_2EZERO})))) (\text{ap} \ \text{c\_2Enum\_2ESUC} \\
& \ V0n)) (\text{ap} \ \text{c\_2Earithmetic\_2ENUMERAL} (\text{ap} \ \text{c\_2Earithmetic\_2EBIT1} \\
& \ \text{c\_2Earithmetic\_2EZERO})))) (\text{ap} (\text{ap} (\text{ap} \ \text{c\_2Ebit\_2EBITS} \ V0n) \ \text{c\_2Enum\_2E0} \\
& \ \ V1a))))))
\end{aligned}$$