

thm_2Ebit_2EDIV__MULT__THM
(TMV8WEB3VB7PRJGm1Q3o1bi7Dvfi3ZorMZB)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap\ P\ x))$ **then** *(the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).*

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A\ P))))$

Definition 4 We define $c_2Ebool_2E_2T$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ P)))$

Definition 6 We define $c_2Ebool_2E_2F$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_2F))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \tag{3}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{5}$$

Definition 10 We define c_Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_Enum_2EABS_num$

Definition 11 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_Enum_2EZERO_REP \in \omega \quad (10)$$

Definition 14 We define c_Enum_2E0 to be $(ap\ c_Enum_2EABS_num\ c_Enum_2EZERO_REP)$.

Definition 15 We define $c_Earithmetic_2EZERO$ to be c_Enum_2E0 .

Definition 16 We define $c_Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_Earithmetic$

Definition 17 We define $c_Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (ap\ (ap\ c_Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_Earithmetic_2E_2B \\ & \quad V1n)\ V0m)))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (ap\ (ap\ c_Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_Earithmetic_2E_2B \\ & \quad V1n)\ V0m)))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad V1n) V2p)) \Rightarrow ((ap (ap c_2Earithmetic_2E_2B V0m) V1n) = V2p) \Leftrightarrow (V0m = \\
& \quad (ap (ap c_2Earithmetic_2E_2D V2p) V1n))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Enum_2Enum. (\forall V1c \in ty_2Enum_2Enum. (\\
& \quad (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2B V0a) \\
& \quad V1c)) V1c) = V0a))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Enum_2Enum. (\forall V1c \in ty_2Enum_2Enum. (\\
& \quad (p (ap (ap c_2Earithmetic_2E_3C_3D V1c) V0b)) \Rightarrow (\forall V2a \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V2a) (ap (ap c_2Earithmetic_2E_2D \\
& \quad V0b) V1c)) = (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2B \\
& \quad V2a) V1c)) V0b))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \Leftrightarrow (\exists V2p \in ty_2Enum_2Enum. \\
& \quad (V1n = (ap (ap c_2Earithmetic_2E_2B V0m) V2p))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad c_2Enum_2E0) V0n)) \Rightarrow (\forall V1k \in ty_2Enum_2Enum. ((V1k = (ap (\\
& \quad ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A (ap (ap c_2Earithmetic_2EDIV \\
& \quad V1k) V0n)) V0n)) (ap (ap c_2Earithmetic_2EMOD V1k) V0n))) \wedge (p (ap \\
& \quad (ap c_2Eprim_rec_2E_3C (ap (ap c_2Earithmetic_2EMOD V1k) V0n)) \\
& \quad V0n))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) \\
& \quad (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL (ap \\
& \quad c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)) V0n))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\text{True} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& \quad True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& \quad (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2) \Rightarrow (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (25)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (26)$$

Theorem 1

$$(\forall V0x \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ (ap\ c_2Earithmetic_2EDIV\ V1n)\ (ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))))\ V0x)))\ (ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))))\ V0x)) = (ap\ (ap\ c_2Earithmetic_2E_2D\ V1n)\ (ap\ (ap\ c_2Earithmetic_2EMOD\ V1n)\ (ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))))\ V0x))))))$$