

thm_2Ebit_2ESLICE__COMP__THM
(TMQCRTVxqLsN3A8Wh5V4SHgwBMSLJ8mahvW)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{2}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \tag{4}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega)^{ty_2Enum_2Enum} \tag{5}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega)^{\omega} \tag{6}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ m))$

Definition 7 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT2 V0n) V1n) = V0n$.

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (8)$$

Definition 9 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Definition 10 We define $c_2Ebit_2ESLICE$ to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.V0h$.

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21) V1t) = V0t$.

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t) = V1t2) V0t1) = V0t1$.

Definition 15 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then $(the (\lambda x.x \in A \Rightarrow P x))$ of type $\iota \Rightarrow \iota$.

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^A)^{27a}).(ap V0P (ap (c_2Emin_2E_40 V0P) V1P) = V0P)$.

Definition 17 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0m$.

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t) = V1t2) V0t1) = V0t1$.

Definition 19 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0m$.

Assume the following.

$$(p (ap (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m) \Rightarrow ((ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2D V0m) V1n)) V1n) = V0m)))) \quad (10)$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Enum_2Enum. (\forall V1b \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Earithmetic_2E_3C_3D V0c) V1b)) \Rightarrow (\forall V2a \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2B V2a) \\
& V1b)) V0c) = (ap (ap c_2Earithmetic_2E_2B V2a) (ap (ap c_2Earithmetic_2E_2D \\
& V1b) V0c))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1h \in ty_2Enum_2Enum. (\\
& \forall V2l \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V2l) V1h)) \Rightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2EMOD \\
& V0n) (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V2l))) (\\
& ap (ap c_2Earithmetic_2EMOD V0n) (ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& (ap c_2Enum_2ESUC V1h))))))
\end{aligned} \tag{12}$$

Assume the following.

$$True \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\
& 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))))
\end{aligned} \tag{19}$$

Theorem 1

$$\begin{aligned} & (\forall V0h \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\ & \quad \forall V2l \in ty_2Enum_2Enum. (\forall V3n \in ty_2Enum_2Enum. ((\\ & \quad (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V1m)) V0h)) \wedge \\ & (p (ap (ap c_2Earithmetic_2E_3C_3D V2l) V1m))) \Rightarrow ((ap (ap c_2Earithmetic_2E_2B \\ & \quad (ap (ap (ap c_2Ebit_2ESLICE V0h) (ap c_2Enum_2ESUC V1m)) V3n)) (\\ & \quad ap (ap (ap c_2Ebit_2ESLICE V1m) V2l) V3n)) = (ap (ap (ap c_2Ebit_2ESLICE \\ & \quad V0h) V2l) V3n)))))) \end{aligned}$$