thm_2Ebitstring_2Ebitstring__nchotomy (TMUxVkoccZG1SRJ8uiDG5L3raHpCxtT9aBG)

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Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \qquad (1)$$

Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota.$

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota.$

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40$\ A$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \qquad (2)$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (\\ ty\_2Ebool\_2Eitself\ A\_27a) \qquad (3)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (4)$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \qquad (5)$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (6)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (7)$$

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Definition 4 *We define* c_2Enum_2E0 *to be* (*ap c_2Enum_2EABS__num c_2Enum_2EZERO__REP*).

Definition 5 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (8)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (9)$$

Definition 6 *We define* c_2Ebool_2ET *to be* (ap (ap (c_2Emin_2E_3D (2^2)) ($\lambda V0x \in 2.V0x$)) ($\lambda V1x \in 2.V1$

Definition 7 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap$ (ap (c_2Emin_2E_3D (2^{A_27}

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap$ $c\_2Enum\_2EABS\_\_num$ (

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(10)$$

Definition 9 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (ap $c\_2Earithmetic\_$

Definition 10 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(11)$$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \qquad (12)$$

Definition 11 *We define* c_2Ebool_2EF *to be* (ap (c_2Ebool_2E_21 2) ($\lambda V0t \in 2.V0t$)).

Definition 12 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 13 *We define* c_2Ebool_2E_7E *to be* ($\lambda V0t \in 2.(ap$ (ap c_2Emin_2E_3D_3D_3E $V0t$) c_2Ebool_2E

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* ($\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) ($\lambda V2t \in$

Definition 15 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 16 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap$ (ap c_2Ebool_2E_2F_5C

Definition 17 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A\_27a : \iota.(ap$ (c_2Emin_2E_40 ($A\_27a^{ty\_2Enum\_2Enu}$

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Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart$$
$$A0\ A1)$$

(13)

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart$$
$$A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)})$$

(14)

Definition 18 *We define* c_2Efcp_2Efcp_index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Let $c\_2Elist\_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EGENLIST\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{ty\_2Enum\_2Enum})^{(A\_27a^{ty\_2Enum\_2Enum})})$$

(15)

Definition 19 *We define* c_2Ebitstring_2Ew2v *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (a$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist$$
$$A\_27a)$$

(16)

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$

(17)

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)})$$

(18)

Let $c\_2Elist\_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETAKE\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum})$$

(19)

Definition 20 *We define* c_2Ebitstring_2Eshiftr *to be* $\lambda V0v \in (ty\_2Elist\_2Elist\ 2).\lambda V1m \in ty\_2Enum\_2En$

Let $c\_2Elist\_2EDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EDROP\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum})$$

(20)

Definition 21 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

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Let $c\_2Elist\_2EAPPEND : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{21}$$

Definition 22 *We define* c$\_2Elist\_2EPAD\_\_LEFT$ *to be* $\lambda A\_27a : \iota.\lambda V0c \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum$

Definition 23 *We define* c$\_2Ebitstring\_2Ezero\_\_extend$ *to be* $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1v \in (ty\_2Elist$

Definition 24 *We define* c$\_2Ebool\_2ECOND$ *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 25 *We define* c$\_2Ebool\_2ELET$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Definition 26 *We define* c$\_2Ebitstring\_2Efixwidth$ *to be* $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1v \in (ty\_2Elist\_2El$

Definition 27 *We define* c$\_2Ebitstring\_2Efield$ *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum$

Definition 28 *We define* c$\_2Ebitstring\_2Etestbit$ *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1v \in (ty\_2Elist\_2Elis$

Definition 29 *We define* c$\_2Efcp\_2EFCP$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap\ ($

Definition 30 *We define* c$\_2Ebitstring\_2Ev2w$ *to be* $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Elist\_2Elist\ 2).(ap\ (c\_2Efcp\_2l$

Definition 31 *We define* c$\_2Ebool\_2E\_5C\_2F$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0w \in (ty\_2Efcp\_2Ecart$$
$$2\ A\_27a).((ap\ (\mathsf{c\_2Ebitstring\_2Ev2w}\ A\_27a)\ (ap\ (\mathsf{c\_2Ebitstring\_2Ew2v} \tag{22}$$
$$A\_27a)\ V0w)) = V0w))$$

Assume the following.

$$True \tag{23}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1){\Rightarrow}(p\ V1t2)){\Rightarrow}(((p$$
$$V1t2){\Rightarrow}(p\ V0t1)){\Rightarrow}((p\ V0t1){\Leftrightarrow}(p\ V1t2)))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Rightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t){\Rightarrow}True){\Leftrightarrow}$$
$$True)\wedge(((False{\Rightarrow}(p\ V0t)){\Leftrightarrow}True)\wedge(((((p\ V0t){\Rightarrow}(p\ V0t)){\Leftrightarrow}True)\wedge((\tag{25}$$
$$(p\ V0t){\Rightarrow}False){\Leftrightarrow}(\neg(p\ V0t)))))))))$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))){\Leftrightarrow}(p\ V0t)))\wedge(((\neg True){\Leftrightarrow}False)\wedge$$
$$((\neg False){\Leftrightarrow}True))) \tag{26}$$

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Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow$$
$$(p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg($$
$$p\ V0t))))))))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{28}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow$$
$$(((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{30}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow$$
$$((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{31}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{32}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ($$
$$(p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg($$
$$p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee$$
$$((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))))$$
$$\tag{33}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ($$
$$(p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge (($$
$$\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))) \tag{34}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{35}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{36}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V0p))))) \tag{37}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q)))))) \qquad (38)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \qquad (39)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart$$
$$2\ A\_27a).(\exists V1v \in (ty\_2Elist\_2Elist\ 2).(V0w = (ap\ \mathsf{c\_2Ebitstring\_2Ev2w}$$
$$A\_27a)\ V1v))))$$