thm_2Ebitstring_2Eextend (TMQgkW68p6mFxD97wk9Lcmg7j5WAVCn6B6v)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{2}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{3}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{4}$$

Definition 5 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Let $c\_2Elist\_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EGENLIST\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{ty\_2Enum\_2Enum})^{(A\_27a^{ty\_2Enum\_2Enum})}) \tag{5}$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{6}$$

Definition 6 *We define* c_2Elist_2EPAD__LEFT *to be* $\lambda A\_27a : \iota.\lambda V0c \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum$

Definition 7 *We define* c_2Ebitstring_2Ezero__extend *to be* $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1v \in (ty\_2Elist\_$

Let $c\_2Elist\_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EHD\ A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{7}$$

Definition 8 *We define* c_2Ebitstring_2Esign__extend *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1v \in ($

Let $c\_2Ebitstring\_2Eextend : \iota$ be given. Assume the following.

$$c\_2Ebitstring\_2Eextend \in ((((ty\_2Elist\_2Elist\ 2)^{(ty\_2Elist\_2Elist\ 2)})^{ty\_2Enum\_2Enum})^{2}) \tag{8}$$

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1l \in (ty\_2Elist\_2Elist\ 2).(\forall V2c \in 2.((ap\ (ap\ (ap\ (\mathsf{c\_2Elist\_2EPAD\_\_LEFT}\ 2)\ V2c)\ V0n)\ V1l) = (ap\ (ap\ (ap\ c\_2Ebitstring\_2Eextend\ V2c)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)\ (ap\ (c\_2Elist\_2ELENGTH\ 2)\ V1l)))\ V1l))))) \tag{9}$$

Assume the following.

$$True \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{13}$$

Theorem 1

$$((\forall V0n \in ty\_2Enum\_2Enum.(\forall V1v \in (ty\_2Elist\_2Elist\ 2).((ap\ (ap\ \mathsf{c\_2Ebitstring\_2Ezero\_\_extend}\ V0n)\ V1v) = (ap\ (ap\ (ap\ c\_2Ebitstring\_2Eextend\ \mathsf{c\_2Ebool\_2EF})\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)\ (ap\ (c\_2Elist\_2ELENGTH\ 2)\ V1v)))\ V1v)))) \wedge (\forall V2n \in ty\_2Enum\_2Enum.(\forall V3v \in (ty\_2Elist\_2Elist\ 2).((ap\ (ap\ (\mathsf{c\_2Ebitstring\_2Esign\_\_extend}\ 2)\ V2n)\ V3v) = (ap\ (ap\ (ap\ c\_2Ebitstring\_2Eextend\ (ap\ (c\_2Elist\_2EHD\ 2)\ V3v))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V2n)\ (ap\ (c\_2Elist\_2ELENGTH\ 2)\ V3v)))\ V3v)))))$$