

thm_2Ebitstring_2Eextend_def_compute (TMWUryS6qDkdbgKqmo9jCmgsdB6z9gRPsJ)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A. \lambda P \in (2^{A \rightarrow 2}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A \rightarrow 2}))\ (\lambda V0P \in 2^{A \rightarrow 2})))$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 6 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT2 n) V0)$

Definition 7 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 8 We define `c_2Earthmetic_2EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earthmetic_2EBIT1\ n)\ V)$

Definition 9 We define `c_2Earithmetic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \text{nonempty } A \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A) \quad (8)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow c_{\text{2Elist_2ECONS}} A_{\text{27a}} \in (((ty_{\text{2Elist_2Elist}} A_{\text{27a}})^{(ty_{\text{2Elist_2Elist}} A_{\text{27a}})})^{A_{\text{27a}}}) \quad (9)$$

Let $c_2Ebitstring_2Eextend : \iota$ be given. Assume the following.

$$c_2Ebitstring_2Eextend \in (((ty_2Elist_2Elist\ 2)^{(ty_2Elist_2Elist\ 2)})^{ty_2Enum_2Enum\ 2})^2) \quad (10)$$

Definition 10 We define $c_{\text{2Emin_2E_3D_3D_3E}}$ to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o} (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Ebool_2E_21 2))(\lambda V2t \in$

Assume the following.

$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0f \in ((A_27a^{\text{ty_2Enum_2Enum}})^{\text{ty_2Enum_2Enum}}).$
 $(\forall V1g \in (A_27a^{\text{ty_2Enum_2Enum}}).((\forall V2n \in \text{ty_2Enum_2Enum}.$
 $((ap\ V1g\ (ap\ c_2Enum_2ESUC\ V2n)) = (ap\ (ap\ V0f\ V2n)\ (ap\ c_2Enum_2ESUC$
 $V2n))) \Leftrightarrow ((\forall V3n \in \text{ty_2Enum_2Enum}.((ap\ V1g\ (ap\ c_2Earthmetic_2ENUMERAL$
 $(ap\ c_2Earthmetic_2EBIT1\ V3n))) = (ap\ (ap\ V0f\ (ap\ (ap\ c_2Earthmetic_2E_2D$
 $(ap\ c_2Earthmetic_2ENUMERAL\ (ap\ c_2Earthmetic_2EBIT1\ V3n))))$
 $(ap\ c_2Earthmetic_2ENUMERAL\ (ap\ c_2Earthmetic_2EBIT1\ c_2Earthmetic_2EZERO))))))$
 $(ap\ c_2Earthmetic_2ENUMERAL\ (ap\ c_2Earthmetic_2EBIT1\ V3n)))))) \wedge$
 $(\forall V4n \in \text{ty_2Enum_2Enum}.((ap\ V1g\ (ap\ c_2Earthmetic_2ENUMERAL$
 $(ap\ c_2Earthmetic_2EBIT2\ V4n))) = (ap\ (ap\ V0f\ (ap\ c_2Earthmetic_2ENUMERAL$
 $(ap\ c_2Earthmetic_2EBIT1\ V4n)))\ (ap\ c_2Earthmetic_2ENUMERAL$
 $(ap\ c_2Earthmetic_2EBIT2\ V4n))))))))))$

Assume the following.

$$\begin{aligned}
 & ((\forall V0v0 \in 2. (\forall V1l \in (ty_2Elist_2Elist 2). ((ap (\\
 & ap (ap c_2Ebitstring_2Eextend V0v0) c_2Enum_2E0) V1l) = V1l))) \wedge \\
 & (\forall V2c \in 2. (\forall V3n \in ty_2Enum_2Enum. (\forall V4l \in (\\
 & ty_2Elist_2Elist 2). ((ap (ap (ap c_2Ebitstring_2Eextend V2c) \\
 & (ap c_2Enum_2ESUC V3n)) V4l) = (ap (ap (ap c_2Ebitstring_2Eextend \\
 & V2c) V3n) (ap (ap (c_2Elist_2ECONS 2) V2c) V4l))))))) \\
 \end{aligned} \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{13}$$

Theorem 1

$$\begin{aligned}
 & ((\forall V0v0 \in 2. (\forall V1l \in (ty_2Elist_2Elist 2). ((ap (\\
 & ap (ap c_2Ebitstring_2Eextend V0v0) c_2Enum_2E0) V1l) = V1l))) \wedge \\
 & (\forall V2c \in 2. (\forall V3n \in ty_2Enum_2Enum. (\forall V4l \in (\\
 & (ty_2Elist_2Elist 2). ((ap (ap (ap c_2Ebitstring_2Eextend V2c) \\
 & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V3n))) \\
 & V4l) = (ap (ap (ap c_2Ebitstring_2Eextend V2c) (ap (ap c_2Earithmetic_2E_2D \\
 & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V3n))) \\
 & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\
 & (ap (ap (c_2Elist_2ECONS 2) V2c) V4l))))))) \wedge (\forall V5c \in 2. (\\
 & \forall V6n \in ty_2Enum_2Enum. (\forall V7l \in (ty_2Elist_2Elist \\
 & 2). ((ap (ap (ap c_2Ebitstring_2Eextend V5c) (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT2 V6n))) V7l) = (ap (ap (ap c_2Ebitstring_2Eextend \\
 & V5c) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
 & V6n))) (ap (ap (c_2Elist_2ECONS 2) V5c) V7l)))))))
 \end{aligned}$$