

thm_2Ebitstring_2En2w_v2n
 (TMbcMSajq4axcPd5MGZ8EZfRhgQv3dcL63C)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. nonempty\ A \Rightarrow nonempty\ (ty_2Elist_2Elist\ A) \quad (2)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \exists a. nonempty\ A \wedge a \in c_2Elist_2ENIL\ A \quad (3)$$

Let $c_2Ebitstring_2Ebitify : \iota$ be given. Assume the following.

$$c_2Ebitstring_2Ebitify \in (((ty_2Elist_2Elist\ ty_2Enum_2Enum)^{(ty_2Elist_2Elist\ 2)})^{(ty_2Elist_2Elist\ ty_2Enum_2Enum)}) \quad (4)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (8)$$

Definition 4 We define c_2Ebool_2ET to be $(ap \ (ap \ (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap \ (ap \ (c_2Emin_2E_3D \ (2^{A_27a})) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x)))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap \ c_2Enum_2EABS_num \ (m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Definition 7 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap \ (ap \ c_2Earithmetic_2E_2B \ (n)) \ (\lambda V1n \in 2.V1n))$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Enumposrep_2El2n : \iota$ be given. Assume the following.

$$c_2Enumposrep_2El2n \in ((ty_2Enum_2Enum^{(ty_2Elist_2Elist \ ty_2Enum_2Enum)})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 9 We define $c_2Enumposrep_2Enum_from_bin_list$ to be $(ap \ c_2Enumposrep_2El2n \ (ap \ c_2Earithmetic_2E_2B \ (c_2Enumposrep_2El2n)))$

Definition 10 We define $c_2Ebitstring_2Ev2n$ to be $\lambda V0l \in (ty_2Elist_2Elist \ 2).(ap \ c_2Enumposrep_2Enum \ (c_2Ebitstring_2Ev2n \ (l)))$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Elist_2ECONS \ A_27a \in (((ty_2Elist_2Elist \ A_27a)^{(ty_2Elist_2Elist \ A_27a)})^{A_27a}) \quad (11)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Elist_2ELENGTH \ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist \ A_27a)}) \quad (12)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Elist_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Elist_2ETAKE \ A_27a \in (((ty_2Elist_2Elist \ A_27a)^{(ty_2Elist_2Elist \ A_27a)})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 11 We define $c_2Ebitstring_2Eshiftr$ to be $\lambda V0v \in (ty_2Elist_2Elist \ 2).\lambda V1m \in ty_2Enum_2Enum.(ap \ c_2Ebitstring_2Ev2n \ (c_2Elist_2ETAKE \ (v)))$

Let $c_2Elist_2EDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Elist_2EDROP \ A_27a \in (((ty_2Elist_2Elist \ A_27a)^{(ty_2Elist_2Elist \ A_27a)})^{ty_2Enum_2Enum}) \quad (15)$$

Definition 12 We define c_Ebool_EF to be $(ap (c_Ebool_E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_Ecombin_EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Let $c_Elist_EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_Elist_EGENLIST A_27a \in (((ty_Elist_Elist A_27a)^{(ty_Enum_Enum)}_{(A_27a^{ty_Enum_Enum})})) \quad (16)$$

Let $c_Elist_EAAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_Elist_EAAPPEND A_27a \in (((ty_Elist_Elist A_27a)^{(ty_Elist_Elist A_27a)}_{(ty_Elist_Elist A_27a)})) \quad (17)$$

Definition 14 We define $c_Elist_EPAD_LEFT$ to be $\lambda A_27a : \iota. \lambda V0c \in A_27a. \lambda V1n \in ty_Enum_Enum. V0c \in V1n$

Definition 15 We define $c_Ebitstring_Ezero_extend$ to be $\lambda V0n \in ty_Enum_Enum. \lambda V1v \in (ty_Elist_Elist A_27a). V0n \in V1v$

Definition 16 We define $c_Emin_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \Rightarrow p Q)$ of type ι .

Definition 17 We define c_Ebool_7E to be $(\lambda V0t \in 2. (ap (ap c_Emin_3D_3D_3E V0t) c_Ebool_2E))$

Definition 18 We define $c_Ebool_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V0t1 \in V2t))))$

Definition 19 We define c_Emin_40 to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge P x) \text{ of type } \iota \Rightarrow \iota)$

Definition 20 We define c_Ebool_3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_Emin_40 V0P) c_Ebool_2E)))$

Definition 21 We define $c_Eprim_rec_3C$ to be $\lambda V0m \in ty_Enum_Enum. \lambda V1n \in ty_Enum_Enum. V0m \in V1n$

Definition 22 We define c_Ebool_ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (V1t1 \in V2t2 \Rightarrow V0t \in V2t2))))$

Definition 23 We define c_Ebool_ELET to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A_27a}). (\lambda V1x \in A_27b. (V0f \in V1x \Rightarrow V0f \in V1x))))$

Definition 24 We define $c_Ebitstring_Efixwidth$ to be $\lambda V0n \in ty_Enum_Enum. \lambda V1v \in (ty_Elist_Elist A_27a). V0n \in V1v$

Definition 25 We define $c_Ebitstring_Efield$ to be $\lambda V0h \in ty_Enum_Enum. \lambda V1l \in ty_Enum_Enum. V0h \in V1l$

Definition 26 We define $c_Ebitstring_Etestbit$ to be $\lambda V0b \in ty_Enum_Enum. \lambda V1v \in (ty_Elist_Elist A_27a). V0b \in V1v$

Let $ty_Efcp_Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_Efcp_Efinite_image A0) \quad (18)$$

Let $ty_Ebool_Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_Ebool_Eitself A0) \quad (19)$$

Let $c_2Ebool_2Ethethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Ebool_2Ethethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (20)$$

Let $c_2Efcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Efcp_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (21)$$

Definition 27 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C))$

Definition 28 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum}))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty } (ty_2Efcp_2Ecart \\ A0 A1) \end{aligned} \quad (22)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Efcp_2Edest_cart \\ A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})(ty_2Efcp_2Ecart A_27a A_27b)) \end{aligned} \quad (23)$$

Definition 29 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in (ty_2Efcp_2Ecart A_27a A_27b)$

Definition 30 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap (c_2Efcp_2Edest_cart A_27a A_27b)))$

Definition 31 We define $c_2Ebitstring_2Ev2w$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2Elist_2Elist 2).(ap (c_2Efcp_2Edest_cart A_27a 2))$

Definition 32 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 33 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a}) A_27a)))$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Elist_2EMAP \\ A_27a A_27b \in (((ty_2Elist_2Elist A_27b)^{(ty_2Elist_2Elist A_27a)}))^{(A_27b^{A_27a})} \end{aligned} \quad (24)$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Elist_2EREVERSE A_27a \in ((ty_2Elist_2Elist \\ A_27a)^{(ty_2Elist_2Elist A_27a)}) \end{aligned} \quad (25)$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (26)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (27)$$

Definition 34 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 35 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 36 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 37 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2E$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (28)$$

Definition 38 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap c_2Enum_2ESUC (ap$

Definition 39 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x.$

Let $c_2Earithmetic_2EEEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (29)$$

Let $c_2Elist_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2EEVERY A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (30)$$

Let $c_2Elist_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2EEL A_27a \in ((A_27a^{(ty_2Elist_2Elist A_27a)})^{ty_2Enum_2Enum}) \quad (31)$$

Definition 40 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (32)$$

Definition 41 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (33)$$

Definition 42 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 43 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum. \lambda V2l \in$

Definition 44 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap$

Definition 45 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 46 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota. \lambda V0n \in ty_2Enum_2Enum. (ap (c_2Efcp_2EFC$

Definition 47 We define $c_2Ewords_2Eword_bit$ to be $\lambda A_27a : \iota. \lambda V0b \in ty_2Enum_2Enum. \lambda V1w \in (ty_2$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))) \\ \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \\ \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p))))))) \\ \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n))))))) \\ \end{aligned} \quad (37)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))))) \\ \end{aligned} \quad (39)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D V0n) c_2Enum_2E0)) \Leftrightarrow (V0n = c_2Enum_2E0))) \quad (40)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE V0m) = \\
 & (ap (ap c_2Earithmetic_2E_2D V0m) (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \\
 \end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
 & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
 & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
 & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\
 & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
 & (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
 & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
 & V0m) V1n))))))) \\
 \end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
 & ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \\
 \end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Eprim_rec_2E_3C \\
 & V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
 & ap (ap c_2Eprim_rec_2E_3C V0m) V2p)))))) \\
 \end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\
 & ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \\
 \end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
 & V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \\
 \end{aligned} \tag{46}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V1n)) V0m))))) \quad (47)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V1n)) V0m))))))) \quad (48)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)) V0n)))) \quad (49)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2D V0m) V1n)) V2p) = (ap (ap c_2Earithmetic_2E_2D V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p))))))) \quad (50)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) (ap (ap c_2Earithmetic_2E_2D V1n) V2p))) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B V0m) V2p)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) c_2Enum_2E0))))))) \quad (51)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C (ap (ap c_2Earithmetic_2E_2D V0m) V1n)) V2p)) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p))) \wedge (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V2p))))))) \quad (52)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}). (\forall V1a \in ty_2Enum_2Enum. (\forall V2b \in ty_2Enum_2Enum. ((p (ap V0P (ap (ap c_2Earithmetic_2E_2D V1a) V2b)) \Leftrightarrow (\forall V3d \in ty_2Enum_2Enum. (((V2b = (ap (ap c_2Earithmetic_2E_2B V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0))) \wedge ((V1a = (ap (ap c_2Earithmetic_2E_2B V2b) V3d)) \Rightarrow (p (ap V0P V3d))))))))))) \quad (53)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in ty_2Enum_2Enum. (\forall V1b \in ty_2Enum_2Enum. (\\
 & (p (ap (ap c_2Earithmetic_2E_3C_3D V0a) V1b)) \Rightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & (ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL (ap \\
 & c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V0a)) (ap (ap \\
 & c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
 & c_2Earithmetic_2EZERO))) V1b)))))) \\
 \end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0i \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & (p (ap (ap c_2Eprim_rec_2E_3C V1n) (ap (ap c_2Earithmetic_2EEEXP \\
 & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
 & V0i))) \Rightarrow (\neg(p (ap (ap c_2Ebit_2EBIT V0i) V1n)))))) \\
 \end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0v \in (ty_2Elist_2Elist 2). (\forall V1a \in (ty_2Elist_2Elist \\
 & ty_2Enum_2Enum). ((ap (ap c_2Ebitstring_2Ebitify V1a) V0v) = (\\
 & ap (ap (c_2Elist_2EAPPEND ty_2Enum_2Enum) (ap (c_2Elist_2EREVERSE \\
 & ty_2Enum_2Enum) (ap (ap (c_2Elist_2EMAP 2 ty_2Enum_2Enum) (\lambda V2b \in \\
 & 2.(ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) V2b) (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) c_2Enum_2E0))) \\
 & V0v))) V1a)))) \\
 \end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0v \in (ty_2Elist_2Elist 2). (p (ap (ap (c_2Elist_2EVERY \\
 & ty_2Enum_2Enum) (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))) (ap (ap \\
 & c_2Ebitstring_2Ebitify (c_2Elist_2ENIL ty_2Enum_2Enum) V0v)))) \\
 \end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
 & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0v \in (ty_2Elist_2Elist \\
 & 2). (\forall V1l \in A_27a. ((ap (c_2Elist_2ELENGTH ty_2Enum_2Enum) \\
 & (ap (ap c_2Ebitstring_2Ebitify (c_2Elist_2ENIL ty_2Enum_2Enum) \\
 & V0v)) = (ap (c_2Elist_2ELENGTH 2) V0v)))) \\
 \end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0b \in ty_2Enum_2Enum. (\forall V1v \in (ty_2Elist_2Elist \\
 & 2).((p (ap (ap c_2Ebitstring_2Etestbit V0b) V1v)) \Leftrightarrow (p (ap (ap (\\
 & c_2Ebool_2ELET ty_2Enum_2Enum 2) (\lambda V2n \in ty_2Enum_2Enum. \\
 & (ap (ap c_2Ebool_2E_2F_5C (ap (ap c_2Eprim_rec_2E_3C V0b) V2n)) \\
 & (ap (ap (c_2Elist_2EEL 2) (ap (ap c_2Earithmetic_2E_2D (ap (ap \\
 & c_2Earithmetic_2E_2D V2n) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
 & c_2Earithmetic_2EZERO)))) V0b)) V1v)))) (ap (c_2Elist_2ELENGTH \\
 & 2) V1v)))))))
 \end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
 & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\
 & \forall V1v \in (ty_2Elist_2Elist 2).((p (ap (ap (c_2Ewords_2Eword_bit \\
 & A_27a) V0n) (ap (c_2Ebitstring_2Ev2w A_27a) V1v))) \Leftrightarrow ((p (ap (ap \\
 & c_2Eprim_rec_2E_3C V0n) (ap (c_2Efcp_2Edimindex A_27a) (c_2Ebool_2Ethe_value \\
 & A_27a)))) \wedge (p (ap (ap c_2Ebitstring_2Etestbit V0n) V1v))))))
 \end{aligned} \tag{60}$$

Assume the following.

$$True \tag{61}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
 V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{62}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{63}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{64}$$

Assume the following.

$$\begin{aligned}
 & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\
 & \forall V0f \in (A_27b^{A_27a}). (\forall V1x \in A_27a. ((ap (ap (c_2Ebool_2ELET \\
 & A_27a A_27b) V0f) V1x) = (ap V0f V1x)))
 \end{aligned} \tag{65}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
 A_27a. (p V0t)) \Leftrightarrow (p V0t))) \tag{66}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge \\
 ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))) \tag{67}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (68)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (69)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (70)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (71)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (72)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (73)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (74)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.((ap(ap(ap(c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap(ap(ap(c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))) \end{aligned} \quad (75)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B)) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (76)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (77)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (78)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (79)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (80)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2))))))) \quad (81)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & V5y_27)))))))))) \end{aligned} \quad (83)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((ap (c_2Ecombin_2EI \\ A_27a) V0x) = V0x)) \quad (84)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \\ & \forall V0x \in (ty_2Efcp_2Ecart A_27a A_27b). (\forall V1y \in (ty_2Efcp_2Ecart \\ & A_27a A_27b). ((V0x = V1y) \Leftrightarrow (\forall V2i \in ty_2Enum_2Enum. ((p (ap \\ & (ap c_2Eprim_rec_2E_3C V2i) (ap (c_2Efcp_2Edimindex A_27b) (\\ & c_2Ebool_2Ethe_value A_27b)))) \Rightarrow ((ap (ap (c_2Efcp_2Efcp_index \\ & A_27a A_27b) V0x) V2i) = (ap (ap (c_2Efcp_2Efcp_index A_27a A_27b) \\ & V1y) V2i))))))) \end{aligned} \quad (85)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\
& \forall V0l \in (ty_2Elist_2Elist A_{27a}).(\forall V1f \in (A_{27b}^{A_{27a}}). \\
& ((ap (c_2Elist_2ELENGTH A_{27b}) (ap (ap (c_2Elist_2EMAP A_{27a} A_{27b}) \\
& V1f) V0l)) = (ap (c_2Elist_2ELENGTH A_{27a}) V0l))) \\
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\
& \forall V0n \in ty_2Enum_2Enum.(\forall V1l \in (ty_2Elist_2Elist \\
& A_{27a}).((p (ap (ap c_2Eprim_rec_2E_3C V0n) (ap (c_2Elist_2ELENGTH \\
& A_{27a}) V1l))) \Rightarrow (\forall V2f \in (A_{27b}^{A_{27a}}).((ap (ap (c_2Elist_2EEL \\
& A_{27b}) V0n) (ap (ap (c_2Elist_2EMAP A_{27a} A_{27b}) V2f) V1l)) = (ap \\
& V2f (ap (ap (c_2Elist_2EEL A_{27a}) V0n) V1l))))))) \\
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& A_{27a}).((ap (c_2Elist_2ELENGTH A_{27a}) (ap (c_2Elist_2EREVERSE \\
& A_{27a}) V0l)) = (ap (c_2Elist_2ELENGTH A_{27a}) V0l))) \\
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0n \in ty_2Enum_2Enum. \\
& \forall V1l \in (ty_2Elist_2Elist A_{27a}).((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) (ap (c_2Elist_2ELENGTH A_{27a}) V1l))) \Rightarrow ((ap (ap (c_2Elist_2EEL \\
& A_{27a}) V0n) (ap (c_2Elist_2EREVERSE A_{27a}) V1l)) = (ap (ap (c_2Elist_2EEL \\
& A_{27a}) (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2D \\
& ap (c_2Elist_2ELENGTH A_{27a}) V1l)) V0n))) V1l)))) \\
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2EiZ (ap \\
& (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& ((\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
(ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
(ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
(ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge (((ap c_2Enum_2ESUC \\
c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
c_2Earithmetic_2EZERO)))) \wedge ((\forall V17n \in ty_2Enum_2Enum. \\
& (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
(ap c_2Enum_2ESUC V17n)))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
(ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
(ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& ((\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
V30m) V29n)))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL \\
V32n)))) \wedge ((\forall V33n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
c_2Enum_2E0) V33n)) \Leftrightarrow False)) \wedge ((\forall V34n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL \\
V34n)) \Leftrightarrow False)))))))
\end{aligned}$$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earthmetic_2EZERO = (ap c_2Earthmetic_2EBIT1 V0n)) \Leftrightarrow False) \wedge \\
& (((ap c_2Earthmetic_2EBIT1 V0n) = c_2Earthmetic_2EZERO) \Leftrightarrow \\
& False) \wedge ((c_2Earthmetic_2EZERO = (ap c_2Earthmetic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap c_2Earthmetic_2EBIT2 V0n) = c_2Earthmetic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap c_2Earthmetic_2EBIT1 V0n) = (ap c_2Earthmetic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c_2Earthmetic_2EBIT2 V0n) = (ap c_2Earthmetic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c_2Earthmetic_2EBIT1 V0n) = (ap c_2Earthmetic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap c_2Earthmetic_2EBIT2 V0n) = (ap c_2Earthmetic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))) \\
\end{aligned} \tag{92}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earthmetic_2EZERO) (ap c_2Earthmetic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earthmetic_2EZERO) \\
& (ap c_2Earthmetic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earthmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earthmetic_2EBIT1 V0n)) (ap c_2Earthmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earthmetic_2EBIT2 V0n)) (ap c_2Earthmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earthmetic_2EBIT1 V0n)) (ap c_2Earthmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg(p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earthmetic_2EBIT2 V0n)) (ap c_2Earthmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m)))))))))))
\end{aligned} \tag{93}$$

Assume the following.

$(\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. ((p (ap (ap c_2Earthmetic_2E_3C_3D c_2Earthmetic_2EZERO) V0n)) \Leftrightarrow True) \wedge (((p (ap (ap c_2Earthmetic_2E_3C_3D (ap c_2Earthmetic_2EBIT1 V0n)) c_2Earthmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earthmetic_2E_3C_3D (ap c_2Earthmetic_2EBIT2 V0n)) c_2Earthmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earthmetic_2E_3C_3D (ap c_2Earthmetic_2EBIT1 V0n)) (ap c_2Earthmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earthmetic_2E_3C_3D V0n) V1m))) \wedge (((p (ap (ap c_2Earthmetic_2E_3C_3D (ap c_2Earthmetic_2EBIT1 V0n)) (ap c_2Earthmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earthmetic_2E_3C_3D V0n) V1m))) \wedge (((p (ap (ap c_2Earthmetic_2E_3C_3D (ap c_2Earthmetic_2EBIT2 V0n)) (ap c_2Earthmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earthmetic_2E_3C_3D V1m) V0n)))) \wedge ((p (ap (ap c_2Earthmetic_2E_3C_3D (ap c_2Earthmetic_2EBIT2 V0n)) (ap c_2Earthmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earthmetic_2E_3C_3D V0n) V1m))))))))))))))$

Assume the following.

$$\begin{aligned}
 & (\forall V0l \in (ty_2Elist_2Elist\ ty_2Enum_2Enum).(\forall V1b \in \\
 & \quad ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) \\
 & \quad V1b)) \Rightarrow (p (ap (ap c_2Eprim_rec_2E_3C (ap (ap c_2Enumposrep_2El2n \\
 & \quad V1b) V0l)) (ap (ap c_2Earithmetic_2EEEXP V1b) (ap (c_2Elist_2ELENGTH \\
 & \quad ty_2Enum_2Enum) V0l))))))) \\
 \end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Enum_2Enum.(\forall V1l \in (ty_2Elist_2Elist \\
 & \quad ty_2Enum_2Enum).(((p (ap (ap (c_2Elist_2EEVERY ty_2Enum_2Enum) \\
 & \quad (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
 & \quad c_2Earithmetic_2EZERO)))) V1l)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\
 & \quad V0x) (ap (c_2Elist_2ELENGTH ty_2Enum_2Enum) V1l)))) \Rightarrow ((p (ap (\\
 & \quad ap c_2Ebit_2EBIT V0x) (ap c_2Enumposrep_2Enum_from_bin_list \\
 & \quad V1l))) \Leftrightarrow ((ap (ap (c_2Elist_2EEL ty_2Enum_2Enum) V0x) V1l) = (ap \\
 & \quad c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \\
 \end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
 & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(
 & \quad \forall V1l1 \in (ty_2Elist_2Elist A_27a).(\forall V2l2 \in (ty_2Elist_2Elist \\
 & \quad A_27a).((p (ap (ap c_2Eprim_rec_2E_3C V0n) (ap (c_2Elist_2ELENGTH \\
 & \quad A_27a) V1l1))) \Rightarrow ((ap (ap (c_2Elist_2EEL A_27a) V0n) (ap (ap (c_2Elist_2EAPPEND \\
 & \quad A_27a) V1l1) V2l2)) = (ap (ap (c_2Elist_2EEL A_27a) V0n) V1l1))))))) \\
 \end{aligned} \tag{97}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{98}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False})) \tag{99}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\
 (((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \tag{100}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\
 ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \tag{101}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \tag{102}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
 & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
 & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))))
 \end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
 & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
 \end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
 & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
 \end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
 & ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
 \end{aligned} \tag{106}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
 (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \tag{107}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \tag{108}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \tag{109}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \tag{110}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \tag{111}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{112}$$

Assume the following.

$$\begin{aligned}
 & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0w \in (ty_2Efcp_2Ecart \\
 & 2 A_{27a}). (\forall V1b \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & V1b) (ap (c_2Efcp_2Edimindex A_{27a}) (c_2Ebool_2Ethe_value A_{27a})))) \Rightarrow \\
 & ((p (ap (ap (c_2Efcp_2Efcp_index 2 A_{27a}) V0w) V1b)) \Leftrightarrow (p (ap \\
 & (c_2Ewords_2Eword_bit A_{27a}) V1b) V0w))))))) \\
 & (113)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0b \in ty_2Enum_2Enum. \\
 & \forall V1n \in ty_2Enum_2Enum. ((p (ap (ap (c_2Ewords_2Eword_bit \\
 & A_{27a}) V0b) (ap (c_2Ewords_2En2w A_{27a}) V1n))) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & V0b) (ap (ap c_2Earithmetic_2E_2D (ap (c_2Efcp_2Edimindex A_{27a}) \\
 & (c_2Ebool_2Ethe_value A_{27a}))) (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \wedge (p (ap \\
 & (ap c_2Ebit_2EBIT V0b) V1n))))))) \\
 & (114)
 \end{aligned}$$

Theorem 1

$$\begin{aligned}
 & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0v \in (ty_2Elist_2Elist \\
 & 2). ((ap (c_2Ewords_2En2w A_{27a}) (ap c_2Ebitstring_2Ev2n V0v)) = \\
 & (ap (c_2Ebitstring_2Ev2w A_{27a}) V0v)))
 \end{aligned}$$