

# thm\_2Ebitstring\_2Eranged\_bitstring\_nchotomy

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NaiqMbBA9Hdm8xaNkm)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1t \in 2.V1t)) (\lambda V2t \in 2.V2t)))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty \ ty\_2Enum\_2Enum \quad (1)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty\_2Elist\_2Elist \ A0) \quad (2)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow c\_2Elist\_2ELENGTH \ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist \ A\_27a)}) \quad (3)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty\_2Ebool\_2Eitself \ A0) \quad (4)$$

Let  $c\_2Ebool\_2Ethethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow c\_2Ebool\_2Ethethe\_value \ A\_27a \in (ty\_2Ebool\_2Eitself \ A\_27a) \quad (5)$$

Let  $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Efcp\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (6)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Elist\_2EDROP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Elist\_2EDROP A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x))$

Let  $c\_2Elist\_2EGENLIST : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Elist\_2EGENLIST A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Enum\_2Enum)})(A\_27a^{ty\_2Enum\_2Enum})) \quad (9)$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Elist\_2EAPPEND A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{(ty\_2Elist\_2Elist A\_27a)}) \quad (10)$$

**Definition 8** We define  $c\_2Elist\_2EPAD\_LEFT$  to be  $\lambda A\_27a : \iota. \lambda V0c \in A\_27a. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 9** We define  $c\_2Ebitstring\_2Ezero\_extend$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. \lambda V1v \in (ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)}$

**Definition 10** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21 2))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (12)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (13)$$

**Definition 11** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num m)$

**Definition 12** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge P x) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a)\ V1x)))$

**Definition 14** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (\lambda V2t \in A\_27a. (V2t = m \vee V2t = n))$

**Definition 15** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. (V2t2 = t1 \wedge V2t2 = t2))))$

**Definition 16** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27b. (V1x = f)))$

**Definition 17** We define  $c\_2Ebitstring\_2Efixwidth$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. \lambda V1v \in (ty\_2Elist\_2Elist\ A\_27a). (\lambda V2v \in (ty\_2Elist\_2Elist\ A\_27a). (V2v = v \wedge V2v = n)))$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in & (ty\_2Elist\_2Elist\ A\_27a) \\ & ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a} \end{aligned} \quad (14)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in & (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \\ & ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a} \end{aligned} \quad (15)$$

Let  $c\_2Elist\_2ETAKE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETAKE\ A\_27a \in & (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \\ & ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum} \end{aligned} \quad (16)$$

**Definition 18** We define  $c\_2Ebitstring\_2Eshiftr$  to be  $\lambda V0v \in (ty\_2Elist\_2Elist\ 2). \lambda V1m \in ty\_2Enum\_2Enum. (\lambda V2m \in A\_27a. (V2m = m \wedge V2m = v)))$

**Definition 19** We define  $c\_2Ebitstring\_2Efield$  to be  $\lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2Enum\_2Enum. (\lambda V2h \in A\_27a. (V2h = h \wedge V2h = l))$

**Definition 20** We define  $c\_2Ebitstring\_2Etestbit$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1v \in (ty\_2Elist\_2Elist\ A\_27a). (\lambda V2v \in (ty\_2Elist\_2Elist\ A\_27a). (V2v = v \wedge V2v = b)))$

Let  $ty\_2Efcp\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_image\ A0) \quad (17)$$

**Definition 21** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Ebool\_2E\_2F\_5C\ A\_27a)\ V1x)\ V1y)))$

**Definition 22** We define  $c\_2Efcp\_2Efinite\_index$  to be  $\lambda A\_27a : \iota. (ap\ (c\_2Emin\_2E\_40\ (A\_27a^{ty\_2Enum\_2Enum}))\ A\_27a)$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \\ & ((ty\_2Efcp\_2Ecart\ A0\ A1)^{(ty\_2Efcp\_2Ecart\ A0\ A1)})^{ty\_2Efcp\_2Ecart\ A0\ A1} \end{aligned} \quad (18)$$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart\ A\_27a\ A\_27b \in & ((A\_27a^{(ty\_2Efcp\_2Efinite\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \\ & ((A\_27a^{(ty\_2Efcp\_2Efinite\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \end{aligned} \quad (19)$$

**Definition 23** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27a). ((ap\ (c\_2Efcp\_2Efcp\_index\ A\_27a)\ V0x) = V0x)$

**Definition 24** We define  $c\_2Efcp\_2EFCP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (ap\ (c\_2Efcp\_2EFCP\ A\_27a)\ V0g) = V0g))$

**Definition 25** We define  $c\_2Ebitstring\_2Ev2w$  to be  $\lambda A\_27a : \iota. \lambda V0v \in (ty\_2Elist\_2Elist\ 2). (ap\ (c\_2Ebitstring\_2Ev2w\ A\_27a)\ V0v) = V0v$

**Definition 26** We define  $c\_2Emarker\_2EAbbrev$  to be  $\lambda V0x \in 2.V0x$ .

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1v \in (ty\_2Elist\_2Elist\ 2). ((ap\ (c\_2Elist\_2ELENGTH\ 2)\ (ap\ (ap\ c\_2Ebitstring\_2Efixwidth\ (V0n)\ V1v)) = V0n)))) \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0v \in (ty\_2Elist\_2Elist\ 2). ((ap\ (c\_2Ebitstring\_2Ev2w\ A\_27a)\ (ap\ (ap\ c\_2Ebitstring\_2Efixwidth\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_value\ A\_27a)))\ V0v)) = (ap\ (c\_2Ebitstring\_2Ev2w\ A\_27a)\ V0v))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a). (\exists V1v \in (ty\_2Elist\_2Elist\ 2). (V0w = (ap\ (c\_2Ebitstring\_2Ev2w\ A\_27a)\ V1v)))) \end{aligned} \quad (22)$$

Assume the following.

$$True \quad (23)$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a). (\exists V1v \in (ty\_2Elist\_2Elist\ 2). ((V0w = (ap\ (c\_2Ebitstring\_2Ev2w\ A\_27a)\ V1v)) \wedge (p\ (ap\ c\_2Emarker\_2EAbbrev\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Elist\_2ELENGTH\ 2)\ V1v))\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_value\ A\_27a)))))))))) \end{aligned}$$