

thm_2Ebitstring_2Ereduce__or__v2w
(TMa4HsGCZk1M1HQkh1enVN9n2nPXTsS7GVn)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ENIL\ A.27a \in (ty_2Elist_2Elist\ A.27a) \quad (2)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2)))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ECONS\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})^{A.27a}) \quad (3)$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \quad (4)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ELENGTH\ A.27a \in (ty_2Eenum_2Eenum^{(ty_2Elist_2Elist\ A.27a)}) \quad (5)$$

Let $c_2Earithmetic_2E2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2D \in ((ty_2Eenum_2Eenum^{ty_2Eenum_2Eenum})^{ty_2Eenum_2Eenum}) \quad (6)$$

Let $c_2Elist_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ETAKE\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})^{ty_2Eenum_2Eenum}) \quad (7)$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Ebitstring_2Eshiftr$ to be $\lambda V0v \in (ty_2Elist_2Elist\ 2). \lambda V1m \in ty_2Enum_2Enum$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (10)$$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Let $c_2Elist_2EDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EDROP\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2)) (\lambda V0t \in 2.V0t)$.

Definition 7 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b.V0x))$

Let $c_2Elist_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EGENLIST\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Enum_2Enum)})^{(A_27a)^{ty_2Enum_2Enum}}) \quad (12)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (13)$$

Definition 8 We define $c_2Elist_2EPAD_LEFT$ to be $\lambda A_27a : \iota. \lambda V0c \in A_27a. \lambda V1n \in ty_2Enum_2Enum$

Definition 9 We define $c_2Ebitstring_2Ezero_extend$ to be $\lambda V0n \in ty_2Enum_2Enum. \lambda V1v \in (ty_2Elist_2Elist$

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t))\ c_2Ebool_2E_21$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21\ 2)) (\lambda V2t \in$

Definition 13 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \setminus p)$ of type $\iota \Rightarrow \iota$).

Definition 14 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 15 We define `c_2Eprim_rec_2E_3C` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define `c_2Ebool_2ECOND` to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 17 We define `c_2Ebool_2ELET` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b$

Definition 18 We define `c_2Ebitstring_2Efixwidth` to be $\lambda V0n \in ty_2Enum_2Enum.\lambda V1v \in (ty_2Elist_2E$

Definition 19 We define `c_2Ebitstring_2Efield` to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum$

Definition 20 We define `c_2Ebitstring_2Etestbit` to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1v \in (ty_2Elist_2E$

Let `ty_2Efcf_2Efinite_image` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Efcf_2Efinite_image A0) \quad (14)$$

Let `ty_2Ebool_2Eitself` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (15)$$

Let `c_2Ebool_2Ethe_value` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (16)$$

Let `c_2Efcf_2Edimindex` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Efcf_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (17)$$

Definition 21 We define `c_2Ebool_2E_3F_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C$

Definition 22 We define `c_2Efcf_2Efinite_index` to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum$

Let `ty_2Efcf_2Ecart` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcf_2Ecart A0 A1) \quad (18)$$

Let `c_2Efcf_2Edest_cart` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcf_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcf_2Efinite_image A_27b)})(ty_2Efcf_2Ecart A_27a A_27b)) \quad (19)$$

Definition 23 We define `c_2Efcf_2Efcf_index` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcf_2Ecart A_27a$

Definition 24 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0g \in (A_27a^{ty_2Enum_2Enum}). (ap$

Definition 25 We define $c_2Ebitstring_2Ev2w$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2Elist_2Elist\ 2). (ap (c_2Efc_2EFCP$

Definition 26 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{20}$$

Definition 27 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 28 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{21}$$

Definition 29 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap\ c_2Earithmetic$

Definition 30 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Elist_2ETL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ETL\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \tag{22}$$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EHD\ A_27a \in (A_27a^{(ty_2Elist_2Elist\ A_27a)}) \tag{23}$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDL\ A_27a\ A_27b \in (((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{(A_27b^{A_27a})^{A_27b}}) \tag{24}$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{25}$$

Definition 31 We define $c_2Ewords_2Eword_reduce$ to be $\lambda A_27a : \iota. \lambda V0f \in ((2^2)^2). \lambda V1w \in (ty_2Efc_2EFCP$

Definition 32 We define $c_2Ewords_2Ereduce_or$ to be $\lambda A_27a : \iota. (ap (c_2Ewords_2Eword_reduce\ A_27a)\ c$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{26}$$

Theorem 1

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0v \in (ty_2Elist_2Elist\ 2). ((ap (c_2Ewords_2Ereduce_or\ A_27a)\ (ap (c_2Ebitstring_2Ev2w\ A_27a)\ V0v)) = (ap (ap (c_2Ewords_2Eword_reduce\ A_27a)\ c_2Ebool_2E_5C_2F)\ (ap (c_2Ebitstring_2Ev2w\ A_27a)\ V0v))))$$