

thm_2Ebitstring_2Etestbit (TMaop-
bCJqwVz42jBXwXDLbc1D8hhRTEiPb1)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40 A$

Definition 4 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (2)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (3)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (4)$$

Definition 7 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Let $c_2Elist_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c_2Elist_2EGENLIST\ A_{27a} \in (((ty_2Elist_2Elist\ A_{27a})^{(ty_2Enum_2Enum)})^{(A_{27a}^{(ty_2Enum_2Enum)})}) \quad (5)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c_2Elist_2EAPPEND\ A_{27a} \in (((ty_2Elist_2Elist\ A_{27a})^{(ty_2Elist_2Elist\ A_{27a})})^{(ty_2Elist_2Elist\ A_{27a})}) \quad (6)$$

Definition 8 We define $c_2Elist_2EPAD_LEFT$ to be $\lambda A_{27a} : \iota. \lambda V0c \in A_{27a}. \lambda V1n \in ty_2Enum_2Enum$

Definition 9 We define $c_2Ebitstring_2Ezero_extend$ to be $\lambda V0n \in ty_2Enum_2Enum. \lambda V1v \in (ty_2Elist_2Elist\ A_{27a})^{(ty_2Elist_2Elist\ A_{27a})}$

Let $c_2Elist_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c_2Elist_2ETAKE\ A_{27a} \in (((ty_2Elist_2Elist\ A_{27a})^{(ty_2Elist_2Elist\ A_{27a})})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 10 We define $c_2Ebitstring_2Eshiftr$ to be $\lambda V0v \in (ty_2Elist_2Elist\ 2). \lambda V1m \in ty_2Enum_2Enum$

Let $c_2Elist_2EDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c_2Elist_2EDROP\ A_{27a} \in (((ty_2Elist_2Elist\ A_{27a})^{(ty_2Elist_2Elist\ A_{27a})})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 11 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \Rightarrow p \ Q)$ of type ι .

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E))$

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V2t1 \Rightarrow V2t2))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (11)$$

Definition 14 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num m)$

Definition 15 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define c_Ebool_ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 17 We define c_Ebool_ELET to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A_27a}). (\lambda V1x \in A_27a. ($

Definition 18 We define $c_Ebitstring_Efixwidth$ to be $\lambda V0n \in ty_2Enum_2Enum. \lambda V1v \in (ty_2Elist_2Elist$

Definition 19 We define $c_Ebitstring_Efield$ to be $\lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist \\ A_27a) \end{aligned} \quad (12)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist \\ A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \end{aligned} \quad (13)$$

Definition 20 We define $c_Ebitstring_Etestbit$ to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1v \in (ty_2Elist_2Elist$

Definition 21 We define $c_Ecombin_ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 22 We define $c_Ecombin_EI$ to be $\lambda A_27a : \iota. (ap (ap (c_Ecombin_ES A_27a (A_27a^{A_27a}) A_27b))$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2EHD A_27a \in (A_27a^{(ty_2Elist_2Elist A_27a)}) \quad (14)$$

Let $c_2Elist_2EEEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2EEEL A_27a \in ((A_27a^{(ty_2Elist_2Elist A_27a)})^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Elist_2EGENLIST_AUX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2EGENLIST_AUX A_27a \in \\ (((((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{ty_2Enum_2Enum})^{(A_27a^{ty_2Enum_2Enum})})) \end{aligned} \quad (16)$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (17)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (18)$$

Definition 23 We define $c_2Earithmetic_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. ($

Definition 24 We define $c_2Ebool_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 25 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (19)$$

Definition 26 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP).$

Definition 27 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2B$

Let $c_2Earithmetic_2EEEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (21)$$

Definition 28 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x.$

Definition 29 We define $c_2Enumeral_2EiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Enum_2ESUC (ap$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (22)$$

Definition 30 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x.$

Definition 31 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EiZ$

Definition 32 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EiZ$

Definition 33 We define $c_2Earithmetic_2EZERO$ to be $c_2Enum_2E0.$

Definition 34 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \quad ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & \quad ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & \quad (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & \quad V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & \quad V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))) \\ & \quad (23) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & \quad V1n) V0m)))) \\ & \quad (24) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\ & (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) V2p))))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\ & (ap c_2Enum_2ESUC V0m)) V1n)))))) \end{aligned} \quad (27)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \\ c_2Enum_2E0) V0n))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\ V1n) V0m))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\ V1n) V0m)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2D (ap c_2Enum_2ESUC V0n)) (ap c_2Enum_2ESUC \\ V1m)) = (ap (ap c_2Earithmetic_2E_2D V0n) V1m)))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2D V0m) V1n) = c_2Enum_2E0) \Leftrightarrow (p (ap (\\ & ap c_2Earithmetic_2E_3C_3D V0m) V1n)))))) \end{aligned} \quad (32)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0m) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\ & (((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\ & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\ & (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\ & (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\ & V0m) V1n))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\ & \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\ & V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\ & ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\ & (p (ap (ap c_2Eprim_rec_2E_3C V1n) V0m)) \Rightarrow (\exists V2p \in ty_2Enum_2Enum. \\ & (V0m = (ap (ap c_2Earithmetic_2E_2B V1n) (ap (ap c_2Earithmetic_2E_2B \\ & V2p) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO))))))) \end{aligned} \quad (36)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2D \\ & V0n) V1m)) V0n)))) \quad (37)$$

Assume the following.

$$(\forall V0a \in ty_2Enum_2Enum. (\forall V1c \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2B V0a) \\ & V1c)) V1c) = V0a))) \quad (38)$$

Assume the following.

$$(\forall V0c \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D V0c) V0c) = c_2Enum_2E0)) \quad (39)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))) \end{aligned} \quad (41)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V1n)) V0m)))))) \quad (42)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V1n)) V0m))))))) \quad (43)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)) V0n)))) \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D V0m) (ap (ap c_2Earithmetic_2E_2D V1n) V2p)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)) V0m) (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2B V0m) V2p)) V1n))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D (ap \\
 & \quad \quad (ap c_2Earithmetic_2E_2D V0m) V1n)) V2p) = (ap (ap c_2Earithmetic_2E_2D \\
 & \quad \quad V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p)))))) \\
 \end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C V0m) \\
 & \quad (ap (ap c_2Earithmetic_2E_2D V1n) V2p))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
 & \quad (ap (ap c_2Earithmetic_2E_2B V0m) V2p)) V1n)))))) \\
 \end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & \quad ap (ap c_2Earithmetic_2E_2D V0m) V1n)) V2p)) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & \quad V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p))) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\
 & \quad c_2Enum_2E0) V2p))))))) \\
 \end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0P \in (2^{ty_2Enum_2Enum}). (\forall V1a \in ty_2Enum_2Enum. \\
 & \quad (\forall V2b \in ty_2Enum_2Enum. ((p (ap V0P (ap (ap c_2Earithmetic_2E_2D \\
 & \quad V1a) V2b))) \Leftrightarrow (\forall V3d \in ty_2Enum_2Enum. (((V2b = (ap (ap c_2Earithmetic_2E_2B \\
 & \quad V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0))) \wedge ((V1a = (ap (ap c_2Earithmetic_2E_2B \\
 & \quad V2b) V3d)) \Rightarrow (p (ap V0P V3d)))))))))) \\
 \end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D (\\
 & \quad ap c_2Enum_2ESUC V0a)) V0a) = (ap c_2Earithmetic_2ENUMERAL (ap \\
 & \quad c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\
 \end{aligned} \tag{50}$$

Assume the following.

$$True \tag{51}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
 \quad V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{52}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{53}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{54}$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\\ \forall V0f \in (A_{27b}^{A_{27a}}).(\forall V1x \in A_{27a}.((ap\ (ap\ (\text{c_2Ebool_2ELET}\ A_{27a}\ A_{27b})\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0t \in 2.((\forall V1x \in A_{27a}.(p\ V0t) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (56)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge (p\ V1t2) \wedge (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (57)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (60)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))) \quad (61)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((V0x = V0x) \Leftrightarrow True)) \quad (62)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1y \in A_{27a}.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (63)$$

Assume the following.

$$((\neg(True \Leftrightarrow False)) \wedge (\neg(False \Leftrightarrow True))) \quad (64)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (65)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. (((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\ & V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (66)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B)))))))))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (69)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (70)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (71)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2))))))) \quad (72)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))))) \end{aligned} \quad (73)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\
& (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\
& V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\
& V5y_27))))))) \\
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a.(\forall V3t2 \in A_27a.((ap \\
& (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V2t1) V3t2) = V3t2)))) \\
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \\
& \forall V0x \in A_27a.(\forall V1y \in A_27b.((ap (ap (c_2Ecombin_2EK \\
& A_27a A_27b) V0x) V1y) = V0x)) \\
\end{aligned} \tag{76}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Ecombin_2EI \\
A_27a) V0x) = V0x)) \tag{77}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0h \in A_27a.(\forall V1t \in \\
& (ty_2Elist_2Elist A_27a).((ap (c_2Elist_2EH D A_27a) (ap (ap (\\
& c_2Elist_2ECONS A_27a) V0h) V1t)) = V0h))) \\
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\
& A_27a).((ap (ap (c_2Elist_2EAPPEND A_27a) (c_2Elist_2ENIL A_27a)) \\
& V0l) = V0l)) \wedge (\forall V1l1 \in (ty_2Elist_2Elist A_27a).(\forall V2l2 \in \\
& (ty_2Elist_2Elist A_27a).(\forall V3h \in A_27a.((ap (ap (c_2Elist_2EAPPEND \\
& A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V3h) V1l1)) V2l2) = (ap (ap \\
& (c_2Elist_2ECONS A_27a) V3h) (ap (ap (c_2Elist_2EAPPEND A_27a) \\
& V1l1) V2l2))))))) \\
\end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (((ap (c_2Elist_2ELENGTH A_27a) \\
& (c_2Elist_2ENIL A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a.(\\
& \forall V1t \in (ty_2Elist_2Elist A_27a).((ap (c_2Elist_2ELENGTH \\
& A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V0h) V1t)) = (ap c_2Enum_2ESUC \\
& (ap (c_2Elist_2ELENGTH A_27a) V1t)))))) \\
\end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist A_{27a})}). \\ & (((p (ap V0P (c_2Elist_2ENIL A_{27a}))) \wedge (\forall V1t \in (ty_2Elist_2Elist A_{27a}).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A_{27a}.(p (ap V0P (ap (ap (c_2Elist_2ECONS A_{27a}) V2h) V1t))))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist A_{27a}).(p (ap V0P V3l)))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0a0 \in A_{27a}.(\forall V1a1 \in (ty_2Elist_2Elist A_{27a}).(\forall V2a0_27 \in A_{27a}.(\forall V3a1_27 \in (ty_2Elist_2Elist A_{27a}).(((ap (ap (c_2Elist_2ECONS A_{27a}) V0a0) V1a1) = (ap (ap (c_2Elist_2ECONS A_{27a}) V2a0_27) V3a1_27)) \Leftrightarrow ((V0a0 = V2a0_27) \wedge (V1a1 = V3a1_27))))))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow \forall A_{27b}. nonempty A_{27b} \Rightarrow (\\ & \forall V0n \in ty_2Enum_2Enum.(\forall V1l \in A_{27b}.(\forall V2ls \in (ty_2Elist_2Elist A_{27b}).(((ap (c_2Elist_2EEEL A_{27a}) c_2Enum_2E0) = (c_2Elist_2EHHD A_{27a})) \wedge ((ap (ap (c_2Elist_2EEEL A_{27b}) (ap c_2Enum_2ESUC V0n)) (ap (ap (c_2Elist_2ECONS A_{27b}) V1l) V2ls)) = (ap (ap (c_2Elist_2EEEL A_{27b}) V0n) V2ls))))))) \end{aligned} \quad (83)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow ((\forall V0n \in ty_2Enum_2Enum. \\ & ((ap (ap (c_2Elist_2EDROP A_{27a}) V0n) (c_2Elist_2ENIL A_{27a})) = (c_2Elist_2ENIL A_{27a}))) \wedge (\forall V1n \in ty_2Enum_2Enum.(\forall V2x \in A_{27a}.(\forall V3xs \in (ty_2Elist_2Elist A_{27a}).((ap (ap (c_2Elist_2EDROP A_{27a}) V1n) (ap (ap (c_2Elist_2ECONS A_{27a}) V2x) V3xs)) = (ap (ap (c_2Ebool_2ECOND (ty_2Elist_2Elist A_{27a})) (ap (ap (c_2Emin_2E_3D ty_2Enum_2Enum) V1n) c_2Enum_2E0)) (ap (ap (c_2Elist_2ECONS A_{27a}) V2x) V3xs)) (ap (ap (c_2Elist_2EDROP A_{27a}) (ap (ap c_2Earithmetic_2E_2D V1n) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \end{aligned} \quad (84)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\forall V1x \in A_{27a}.(\forall V2xs \in (ty_2Elist_2Elist A_{27a}).(\\ & (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)) \Rightarrow ((ap (ap (c_2Elist_2ETAKE A_{27a}) V0n) (ap (ap (c_2Elist_2ECONS A_{27a}) V1x) V2xs)) = (ap (ap (c_2Elist_2ECONS A_{27a}) V1x) (ap (ap (c_2Elist_2ETAKE A_{27a}) (ap (ap c_2Earithmetic_2E_2D V0n) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \end{aligned} \quad (85)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0l \in (ty_2Elist_2Elist \\ A_{27a}).((ap\ (ap\ (c_2Elist_2ETAKE\ A_{27a})\ c_2Enum_2E0)\ V0l) = (c_2Elist_2ENIL \\ A_{27a}))) \end{aligned} \quad (86)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0n \in ty_2Enum_2Enum.(\\ \forall V1l \in (ty_2Elist_2Elist\ A_{27a}).((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ V0n)\ (ap\ (c_2Elist_2ELENGTH\ A_{27a})\ V1l))) \Rightarrow ((ap\ (c_2Elist_2ELENGTH \\ A_{27a})\ (ap\ (ap\ (c_2Elist_2ETAKE\ A_{27a})\ V0n)\ V1l)) = V0n)))) \end{aligned} \quad (87)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & ((\forall V0f \in (A_{27a}^{ty_2Enum_2Enum}). \\ (\forall V1l \in (ty_2Elist_2Elist\ A_{27a}).((ap\ (ap\ (ap\ (c_2Elist_2EGENLIST_AUX \\ A_{27a})\ V0f)\ c_2Enum_2E0)\ V1l)) = V1l))) \wedge ((\forall V2f \in (A_{27a}^{ty_2Enum_2Enum}). \\ (\forall V3n \in ty_2Enum_2Enum.(\forall V4l \in (ty_2Elist_2Elist \\ A_{27a}).((ap\ (ap\ (ap\ (c_2Elist_2EGENLIST_AUX\ A_{27a})\ V2f)\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT1\ V3n)))\ V4l) = (ap\ (ap\ (ap\ (c_2Elist_2EGENLIST_AUX \\ A_{27a})\ V2f)\ (ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT1\ V3n)))\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))\ (ap\ (ap \\ (c_2Elist_2ECONS\ A_{27a})\ (ap\ V2f\ (ap\ (ap\ c_2Earithmetic_2E_2D \\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ V3n))))\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))) \wedge (\forall V5f \in (A_{27a}^{ty_2Enum_2Enum}).(\forall V6n \in \\ ty_2Enum_2Enum.(\forall V7l \in (ty_2Elist_2Elist\ A_{27a}).((ap \\ (ap\ (ap\ (c_2Elist_2EGENLIST_AUX\ A_{27a})\ V5f)\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT2\ V6n)))\ V7l) = (ap\ (ap\ (ap\ (c_2Elist_2EGENLIST_AUX \\ A_{27a})\ V5f)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\ V6n)))\ (ap\ (ap\ (c_2Elist_2ECONS\ A_{27a})\ (ap\ V5f\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT1\ V6n))))\ V7l))))))) \end{aligned} \quad (88)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0f \in (A_{27a}^{ty_2Enum_2Enum}). \\ (\forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ (c_2Elist_2EGENLIST\ A_{27a}) \\ V0f)\ c_2Enum_2E0) = (c_2Elist_2ENIL\ A_{27a})) \wedge ((ap\ (ap\ (c_2Elist_2EGENLIST \\ A_{27a})\ V0f)\ (ap\ c_2Earithmetic_2ENUMERAL\ V1n)) = (ap\ (ap\ (ap\ (c_2Elist_2EGENLIST_AUX \\ A_{27a})\ V0f)\ (ap\ c_2Earithmetic_2ENUMERAL\ V1n))\ (c_2Elist_2ENIL \\ A_{27a}))))))) \end{aligned} \quad (89)$$

Assume the following.

$(\forall V0n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0n) = V0n)) \wedge (\forall V1n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge (\forall V2n \in ty_2Enum_2Enum. (\forall V3m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V3m) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2EiZ (ap (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge (\forall V4n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge (\forall V5n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge (\forall V6n \in ty_2Enum_2Enum. (\forall V7m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A V6n) V7m)))))) \wedge (\forall V8n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge (\forall V9n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge (\forall V10n \in ty_2Enum_2Enum. (\forall V11m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V10n) V11m)))))) \wedge (\forall V12n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V12n)))) = c_2Enum_2E0)) \wedge (\forall V13n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V13n)))) = c_2Enum_2E0)) \wedge (\forall V14n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEEXP V14n) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge (\forall V15n \in ty_2Enum_2Enum. (\forall V16m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL V15n)) (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2EEEXP V15n) V16m)))))) \wedge (((ap c_2Enum_2ESUC c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge (\forall V17n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2ESUC V17n)))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V18n \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge (\forall V19n \in ty_2Enum_2Enum. (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge (\forall V20n \in ty_2Enum_2Enum. ((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge (\forall V21n \in ty_2Enum_2Enum. ((\forall V22m \in ty_2Enum_2Enum. (((ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge ((\forall V23n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C V23n) c_2Enum_2E0)) \Leftrightarrow False))) \wedge (\forall V24n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum. (\forall V26m \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C V25n) c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V26m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V27n)) \Leftrightarrow False))) \wedge ((\forall V28n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3E V28n) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3E V29n) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V29n)))))) \wedge ((\forall V30m \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V30m)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V30m)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V31n)) \Leftrightarrow True))) \wedge ((\forall V32n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V32n)) \Leftrightarrow False))) \wedge ((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V32n)) \Leftrightarrow False))$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& (ap c_2Earithmetic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg(p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m)))))))))) \\
\end{aligned} \tag{92}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m)))))))))) \\
\end{aligned} \tag{93}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{94}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{95}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \tag{96}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \tag{97}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (98)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (99)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & ((\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (100)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (101)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (102)$$

Theorem 1

$$\begin{aligned} & (\forall V0b \in ty_2Enum_2Enum.(\forall V1v \in (ty_2Elist_2Elist \\ & 2).((p (ap (ap c_2Ebitstring_2Etestbit V0b) V1v)) \Leftrightarrow (p (ap (ap (\\ & c_2Ebool_2ELET ty_2Enum_2Enum 2) (\lambda V2n \in ty_2Enum_2Enum. \\ & (ap (ap c_2Ebool_2E_2F_5C (ap (ap c_2Eprim_rec_3C V0b) V2n)) \\ & (ap (ap (c_2Elist_2EEL 2) (ap (ap c_2Earithmetic_2E_2D (ap (ap \\ & c_2Earithmetic_2E_2D V2n) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))) V0b)) V1v)))) (ap (c_2Elist_2ELENGTH \\ & 2) V1v))))))) \end{aligned}$$