

thm\_2Ebitstring\_2Ew2w\_v2w  
 (TMUV7SmaeSrxbXnL18JH87UZujiw2hK9HHF)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (2)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (3)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Elist\_2EDROP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2EDROP\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (5)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ (\lambda V1P \in 2.V1P))\ (\lambda V2P \in 2.V2P)))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x))$

Let  $c\_2Elist\_2EGENLIST : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c\_2Elist\_2EGENLIST\ A_{27a} \in (((ty\_2Elist\_2Elist\ A_{27a})^{ty\_2Enum\_2Enum})^{(A_{27a}^{ty\_2Enum\_2Enum})}) \quad (6)$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c\_2Elist\_2EAPPEND\ A_{27a} \in (((ty\_2Elist\_2Elist\ A_{27a})^{(ty\_2Elist\_2Elist\ A_{27a})})^{(ty\_2Elist\_2Elist\ A_{27a})}) \quad (7)$$

**Definition 6** We define  $c\_2Elist\_2EPAD\_LEFT$  to be  $\lambda A_{27a} : \iota. \lambda V0c \in A_{27a}. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 7** We define  $c\_2Ebitstring\_2Ezero\_extend$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. \lambda V1v \in (ty\_2Elist\_2Elist\ A_{27a})$

**Definition 8** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF))$

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V2t) c\_2Ebool\_2EF)))))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (10)$$

**Definition 11** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num m)$

**Definition 12** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x)) \text{ else } (\lambda x. x \in A \wedge \neg p x)$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (ap V0P (ap (c\_2Emin\_2E\_40 A_{27a}) P)))$

**Definition 14** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 15** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A_{27a} : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_{27a}. (\lambda V2t2 \in A_{27a}. (ap (ap c\_2Emin\_2E\_40 A_{27a}) t1) t2)))$

**Definition 16** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A_{27a} : \iota. \lambda A_{27b} : \iota. (\lambda V0f \in (A_{27b}^{A_{27a}}). (\lambda V1x \in A_{27b}. (ap (ap c\_2Emin\_2E\_40 A_{27b}) f) x)))$

**Definition 17** We define  $c\_2Ebitstring\_2Efixwidth$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. \lambda V1v \in (ty\_2Elist\_2Elist\ A_{27a})$

Let  $c\_2Elist\_2EEL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c\_2Elist\_2EEL\ A_{27a} \in ((A_{27a})^{(ty\_2Elist\_2Elist\ A_{27a})})^{ty\_2Enum\_2Enum} \quad (11)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c\_2Elist\_2ENIL\ A_{27a} \in (ty\_2Elist\_2Elist\ A_{27a}) \quad (12)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c\_2Elist\_2ECONS\ A_{27a} \in (((ty\_2Elist\_2Elist\ A_{27a})^{(ty\_2Elist\_2Elist\ A_{27a})})^{A_{27a}}) \quad (13)$$

Let  $c\_2Elist\_2ETAKE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c\_2Elist\_2ETAKE\ A_{27a} \in (((ty\_2Elist\_2Elist\ A_{27a})^{(ty\_2Elist\_2Elist\ A_{27a})})^{ty\_2Enum\_2Enum}) \quad (14)$$

**Definition 18** We define  $c\_2Ebitstring\_2Eshiftr$  to be  $\lambda V0v \in (ty\_2Elist\_2Elist\ 2).\lambda V1m \in ty\_2Enum\_2Enum$

**Definition 19** We define  $c\_2Ebitstring\_2Efield$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum$

**Definition 20** We define  $c\_2Ebitstring\_2Etestbit$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1v \in (ty\_2Elist\_2Elist\ 2)$

Let  $ty\_2Efcp\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_image\ A0) \quad (15)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (16)$$

Let  $c\_2Ebool\_2Ethethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c\_2Ebool\_2Ethethe\_value\ A_{27a} \in (ty\_2Ebool\_2Eitself\ A_{27a}) \quad (17)$$

Let  $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c\_2Efcp\_2Edimindex\ A_{27a} \in (ty\_2Enum\_2Enum)^{(ty\_2Ebool\_2Eitself\ A_{27a})} \quad (18)$$

**Definition 21** We define  $c\_2Ebool\_2E_3F_21$  to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap\ (ap\ c\_2Ebool\_2E_2F_5C$

**Definition 22** We define  $c\_2Efcp\_2Efinite\_index$  to be  $\lambda A_{27a} : \iota.(ap\ (c\_2Emin\_2E_40\ (A_{27a})^{ty\_2Enum\_2Enum}))$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_0.nonempty \ A_0 \Rightarrow & \forall A_1.nonempty \ A_1 \Rightarrow nonempty (ty\_2Efcp\_2Ecart \\ & A_0 \ A_1) \end{aligned} \quad (19)$$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty \ A_{27a} \Rightarrow & \forall A_{27b}.nonempty \ A_{27b} \Rightarrow c\_2Efcp\_2Edest\_cart \\ & A_{27a} \ A_{27b} \in ((A_{27a}^{(ty\_2Efcp\_2Efinit\_image \ A_{27b})})^{(ty\_2Efcp\_2Ecart \ A_{27a} \ A_{27b})}) \end{aligned} \quad (20)$$

**Definition 23** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A_{27a} : \iota. \lambda A_{27b} : \iota. \lambda V0x \in (ty\_2Efcp\_2Ecart \ A_{27a} \ A_{27b})$

**Definition 24** We define  $c\_2Efcp\_2EFCP$  to be  $\lambda A_{27a} : \iota. \lambda A_{27b} : \iota. (\lambda V0g \in (A_{27a}^{ty\_2Enum\_2Enum}).(ap \ (c\_2Efcp\_2Efcp\_index \ A_{27a} \ A_{27b}) \ g))$

**Definition 25** We define  $c\_2Ebitstring\_2Ev2w$  to be  $\lambda A_{27a} : \iota. \lambda V0v \in (ty\_2Elist\_2Elist \ 2).(ap \ (c\_2Efcp\_2Efcp\_index \ A_{27a}) \ v)$

**Definition 26** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A_{27a} : \iota. \lambda A_{27b} : \iota. \lambda A_{27c} : \iota. (\lambda V0f \in ((A_{27c}^{A_{27b}})^{A_{27a}}).f)$

**Definition 27** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A_{27a} : \iota. (ap \ (ap \ (c\_2Ecombin\_2ES \ A_{27a}) \ (A_{27a}^{A_{27a}}) \ A_{27a}))$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (21)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (22)$$

**Definition 28** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. m \neq n$

**Definition 29** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap \ (c\_2Ebool\_2E\_21 \ 2) \ (\lambda V2t \in 2. t1 \neq t2))))$

**Definition 30** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. m = n$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (23)$$

**Definition 31** We define  $c\_2Enum\_2E0$  to be  $(ap \ c\_2Enum\_2EABS\_num \ c\_2Enum\_2EZERO\_REP)$

**Definition 32** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap \ (ap \ (ap \ (c\_2Ebool\_2E\_21 \ 2) \ (c\_2Eprim\_rec \ 2))) \ m))$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (24)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (25)$$

**Definition 33** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

**Definition 34** We define  $c\_2Enumeral\_2EiiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2ESUC (ap$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (26)$$

**Definition 35** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

**Definition 36** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic$

**Definition 37** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic$

**Definition 38** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 39** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 40** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2. \lambda V1n \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Eboo$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (27)$$

**Definition 41** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart 2 A\_27a). (ap (ap c\_2Ebit\_2EDIV : \iota$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (28)$$

**Definition 42** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (29)$$

**Definition 43** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 44** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2Enum\_2Enum. \lambda V2m \in ty\_2Enum\_2Enum$

**Definition 45** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap (ap c\_2Ebit\_2EDIV : \iota$

**Definition 46** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_2Enum\_2Enum. (ap (c\_2Efcp\_2EFC$

**Definition 47** We define  $c\_2Ewords\_2Ew2w$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart 2 A\_27b). (ap (c\_2Ebit\_2EDIV : \iota$

**Definition 48** We define  $c\_2Ewords\_2Eword\_bit$  to be  $\lambda A\_27a : \iota. \lambda V0b \in ty\_2Enum\_2Enum. \lambda V1w \in (ty\_2Efcp\_2Ecart 2 A\_27a). (ap (c\_2Ebit\_2EDIV : \iota$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\
& ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\
& V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& V1n) V0m)))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& V1n) V0m)))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
& (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Enum\_2ESUC V0m)) V1n))))
\end{aligned} \tag{34}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V0n))) \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1n) V0m))))
\end{aligned} \tag{36}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
V0n) c\_2Enum\_2E0)) \Leftrightarrow (V0n = c\_2Enum\_2E0))) \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge \\
& ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
& (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V0m) V1n)))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
& ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p ( \\
& ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Enum\_2ESUC V1n)) V0m))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (\neg(V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& V1n)) V0m))))))
\end{aligned} \tag{43}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0n))) \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2D (ap (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p) = (ap (ap c\_2Earithmetic\_2E\_2D V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) (ap (ap c\_2Earithmetic\_2E\_2D V1n) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) c\_2Enum\_2E0))))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p)) \Leftrightarrow ((p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p))) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V2p))))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (\forall V1a \in ty\_2Enum\_2Enum. \\ & (\forall V2b \in ty\_2Enum\_2Enum. ((p (ap V0P (ap (ap c\_2Earithmetic\_2E\_2D V1a) V2b))) \Leftrightarrow (\forall V3d \in ty\_2Enum\_2Enum. (((V2b = (ap (ap c\_2Earithmetic\_2E\_2B V1a) V3d)) \Rightarrow (p (ap V0P c\_2Enum\_2E0))) \wedge ((V1a = (ap (ap c\_2Earithmetic\_2E\_2B V2b) V3d)) \Rightarrow (p (ap V0P V3d)))))))))) \end{aligned} \quad (49)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1v \in (ty\_2Elist\_2Elist 2). ((ap (c\_2Elist\_2ELENGTH 2) (ap (ap c\_2Ebitstring\_2Efixwidth V0n) V1v)) = V0n))) \quad (50)$$

Assume the following.

$$\begin{aligned}
& (\forall V0i \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& \forall V2w \in (ty\_2Elist\_2Elist 2). ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0i) V1n)) \Rightarrow ((p (ap (ap (c\_2Elist\_2EEL 2) V0i) (ap (ap c\_2Ebitstring\_2Efixwidth \\
& V1n) V2w))) \Leftrightarrow (p (ap (ap (ap (c\_2Ebool\_2ECOND 2) (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap (c\_2Elist\_2ELENGTH 2) V2w)) V1n)) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2D \\
& V1n) (ap (c\_2Elist\_2ELENGTH 2) V2w))) V0i)) (ap (ap (c\_2Elist\_2EEL \\
& 2) (ap (ap c\_2Earithmetic\_2E\_2D V0i) (ap (ap c\_2Earithmetic\_2E\_2D \\
& V1n) (ap (c\_2Elist\_2ELENGTH 2) V2w)))) V2w))) (ap (ap (c\_2Elist\_2EEL \\
& 2) (ap (ap c\_2Earithmetic\_2E\_2B V0i) (ap (ap c\_2Earithmetic\_2E\_2D \\
& (ap (c\_2Elist\_2ELENGTH 2) V2w)) V1n))) V2w))))))) \\
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty\_2Enum\_2Enum. (\forall V1v \in (ty\_2Elist\_2Elist \\
& 2). ((p (ap (ap c\_2Ebitstring\_2Etestbit V0b) V1v)) \Leftrightarrow (p (ap (ap ( \\
& c\_2Ebool\_2ELET ty\_2Enum\_2Enum 2) (\lambda V2n \in ty\_2Enum\_2Enum. \\
& (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap c\_2Eprim\_rec\_2E\_3C V0b) V2n)) \\
& (ap (ap (c\_2Elist\_2EEL 2) (ap (ap c\_2Earithmetic\_2E\_2D (ap (ap \\
& c\_2Earithmetic\_2E\_2D V2n) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO)))) V0b)) V1v)))) (ap (c\_2Elist\_2ELENGTH \\
& 2) V1v))))))) \\
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum. \\
& \forall V1v \in (ty\_2Elist\_2Elist 2). ((p (ap (ap (c\_2Ewords\_2Eword\_bit \\
& A\_27a) V0n) (ap (c\_2Ebitstring\_2Ev2w A\_27a) V1v))) \Leftrightarrow ((p (ap (ap \\
& c\_2Eprim\_rec\_2E\_3C V0n) (ap (c\_2Efcp\_2Edimindex A\_27a) (c\_2Ebool\_2Ethe\_value \\
& A\_27a)))) \wedge (p (ap (ap c\_2Ebitstring\_2Etestbit V0n) V1v)))))) \\
\end{aligned} \tag{53}$$

Assume the following.

$$True \tag{54}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{55}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{56}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{57}$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & \forall V0f \in (A_{27b}^{A_{27a}}).(\forall V1x \in A_{27a}.((ap (ap (c_2Ebool\_2ELET \\ & A_{27a} A_{27b}) V0f) V1x) = (ap V0f V1x))) \end{aligned} \quad (58)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_{27a}.(p V0t)) \Leftrightarrow (p V0t))) \quad (59)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge \\ ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow \\ & (p V0t)) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (63)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))) \quad (64)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((V0x = V0x) \Leftrightarrow \\ True)) \quad (65)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1y \in A_{27a}.((V0x = V1y) \Leftrightarrow \\ (V1y = V0x)))) \quad (66)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \\ & V0t)))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0t1 \in A_{27a}.(\forall V1t2 \in \\ A_{27a}.(((ap\ (ap\ (ap\ (c_{2Ebool\_2ECOND}\ A_{27a})\ c_{2Ebool\_2ET})\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_{2Ebool\_2ECOND}\ A_{27a})\ c_{2Ebool\_2EF}) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (68)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ( \\ (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C))))))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.((((\neg(p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \\ \vee (\neg(p\ V1B)))) \wedge ((\neg(p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge (\neg(p\ V1B)))))))))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \\ \vee (p\ V1B)))))) \quad (71)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (72)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \\ \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3))))))) \quad (73)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p\ V0t1) \\ \wedge (p\ V1t2)) \vee ((\neg(p\ V0t1)) \wedge (\neg(p\ V1t2))))))) \quad (74)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in \\ 2.(((p\ V0x) \Leftrightarrow (p\ V1x_{27})) \wedge ((p\ V1x_{27}) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_{27})))))) \Rightarrow \quad (75) \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_{27}) \Rightarrow (p\ V3y_{27})))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_{27a}.(\forall V3x_{27} \in A_{27a}.(\forall V4y \in A_{27a}. \\ & (\forall V5y_{27} \in A_{27a}.(((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_{27})) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_{27})))) \Rightarrow ((ap\ (ap\ (ap\ (c_{2Ebool\_2ECOND}\ A_{27a})\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_{2Ebool\_2ECOND}\ A_{27a})\ V1Q)\ V3x_{27}) \\ & V5y_{27})))))))))) \end{aligned} \quad (76)$$

Assume the following.

$$\begin{aligned}
 & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\forall V0t1 \in A_{27a}. (\forall V1t2 \in \\
 & A_{27a}. ((ap (ap (ap (c_2Ebool_2ECOND A_{27a}) c_2Ebool_2ET) V0t1) \\
 & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{27a}. (\forall V3t2 \in A_{27a}. ((ap \\
 & (ap (c_2Ebool_2ECOND A_{27a}) c_2Ebool_2EF) V2t1) V3t2) = V3t2)))) \\
 & (77)
 \end{aligned}$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. ((ap (c_2Ecombin_2EI \\
 A_{27a}) V0x) = V0x)) \quad (78)$$

Assume the following.

$$\begin{aligned}
 & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow \\
 & \forall V0x \in (ty\_2Efcp\_2Ecart A_{27a} A_{27b}). (\forall V1y \in (ty\_2Efcp\_2Ecart \\
 & A_{27a} A_{27b}). ((V0x = V1y) \Leftrightarrow (\forall V2i \in ty\_2Enum\_2Enum. ((p (ap \\
 & (ap c_2Eprim\_rec_2E_3C V2i) (ap (c_2Efcp\_2Edimindex A_{27b}) ( \\
 & c_2Ebool_2Ethe\_value A_{27b})))) \Rightarrow ((ap (ap (c_2Efcp\_2Efcp\_index \\
 & A_{27a} A_{27b}) V0x) V2i) = (ap (ap (c_2Efcp\_2Efcp\_index A_{27a} A_{27b}) \\
 & V1y) V2i))))))) \\
 & (79)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enum\_2EiZ (ap \\
& (ap c\_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& ((\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
(ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge (((ap c\_2Enum\_2ESUC \\
c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
c\_2Earithmetic\_2EZERO)))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. \\
& (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Enum\_2ESUC V17n)))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
(ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL \\
V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& ((\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V30m) V29n)))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V32n)))) \wedge ((\forall V33n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V33n)) \Leftrightarrow False)) \wedge ((\forall V34n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V34n)) \Leftrightarrow False)))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. (\forall V0n \in ty\_2Enum\_2Enum. \\
& ((ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B c\_2Earithmetic\_2ZERO) \\
& V0n)) = V0n) \wedge (((ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0n) c\_2Earithmetic\_2ZERO)) = V0n) \wedge (((ap c\_2Enumeral\_2EiZ ( \\
& ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2EBIT1 V0n)) ( \\
& ap c\_2Earithmetic\_2EBIT1 V1m))) = (ap c\_2Earithmetic\_2EBIT2 ( \\
& ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B V0n) V1m)))) \wedge \\
& (((ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0n) V1m)))) \wedge \\
& ((ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0n) V1m)))) \wedge \\
& ((ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) = (ap c\_2Earithmetic\_2EBIT2 \\
& (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0n) V1m)))) \wedge \\
& ((ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B c\_2Earithmetic\_2ZERO) \\
& V0n)) = (ap c\_2Enum\_2ESUC V0n) \wedge (((ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0n) c\_2Earithmetic\_2ZERO)) = (ap c\_2Enum\_2ESUC V0n)) \wedge (((ap \\
& c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0n) V1m)))) \wedge \\
& ((ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) = (ap c\_2Earithmetic\_2EBIT2 \\
& (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0n) V1m)))) \wedge \\
& ((ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) = (ap c\_2Earithmetic\_2EBIT2 \\
& (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0n) V1m)))) \wedge \\
& ((ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap c\_2Enumeral\_2EiiSUC (ap (ap c\_2Earithmetic\_2E\_2B V0n) V1m)))) \wedge \\
& (((ap c\_2Enumeral\_2EiiSUC (ap (ap c\_2Earithmetic\_2E\_2B c\_2Earithmetic\_2ZERO) \\
& V0n)) = (ap c\_2Enumeral\_2EiiSUC V0n) \wedge (((ap c\_2Enumeral\_2EiiSUC \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0n) c\_2Earithmetic\_2ZERO)) = ( \\
& ap c\_2Enumeral\_2EiiSUC V0n)) \wedge (((ap c\_2Enumeral\_2EiiSUC (ap ( \\
& ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT1 \\
& V1m))) = (ap c\_2Earithmetic\_2EBIT2 (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0n) V1m)))) \wedge (((ap c\_2Enumeral\_2EiiSUC (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) = \\
& (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Enumeral\_2EiiSUC (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0n) V1m)))) \wedge (((ap c\_2Enumeral\_2EiiSUC (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) = \\
& (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Enumeral\_2EiiSUC (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0n) V1m)))) \wedge (((ap c\_2Enumeral\_2EiiSUC (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) = \\
& (ap c\_2Earithmetic\_2EBIT2 (ap c\_2Enumeral\_2EiiSUC (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0n) V1m))))))))))))))))))))))) \\
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{82}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{83}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
\end{aligned} \tag{86}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{87}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p \\
& V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \\
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\ & \end{aligned} \quad (90)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (91)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) \\ & (ap (c\_2Efcp\_2Edimindex A\_27a) (c\_2Ebool\_2Ethet\_value A\_27a)))) \quad (92) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \\ & \forall V0w \in (ty\_2Efcp\_2Ecart 2 A\_27a). (\forall V1i \in ty\_2Enum\_2Enum. \\ & ((p (ap (ap c\_2Eprim\_rec\_2E\_3C V1i) (ap (c\_2Efcp\_2Edimindex A\_27b) \\ & (c\_2Ebool\_2Ethet\_value A\_27b)))) \Rightarrow ((p (ap (ap (c\_2Efcp\_2Efcp\_index \\ & 2 A\_27b) (ap (c\_2Ewords\_2Ew2w A\_27a A\_27b) V0w)) V1i)) \Leftrightarrow ((p (ap \\ & (ap c\_2Eprim\_rec\_2E\_3C V1i) (ap (c\_2Efcp\_2Edimindex A\_27a) ( \\ & c\_2Ebool\_2Ethet\_value A\_27a)))) \wedge (p (ap (ap (c\_2Efcp\_2Efcp\_index \\ & 2 A\_27a) V0w) V1i))))))) \quad (93) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart \\ & 2 A\_27a). (\forall V1b \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\ & V1b) (ap (c\_2Efcp\_2Edimindex A\_27a) (c\_2Ebool\_2Ethet\_value A\_27a)))) \Rightarrow \\ & ((p (ap (ap (c\_2Efcp\_2Efcp\_index 2 A\_27a) V0w) V1b)) \Leftrightarrow (p (ap \\ & (c\_2Ewords\_2Eword\_bit A\_27a) V1b) V0w))))))) \quad (94) \end{aligned}$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \\ & \forall V0v \in (ty\_2Elist\_2Elist 2). ((ap (c\_2Ewords\_2Ew2w A\_27a \\ & A\_27b) (ap (c\_2Ebitstring\_2Ev2w A\_27a) V0v)) = (ap (c\_2Ebitstring\_2Ev2w \\ & A\_27b) (ap (ap c\_2Ebitstring\_2Efixwidth (ap (ap (ap (c\_2Ebool\_2ECOND \\ & ty\_2Enum\_2Enum) (ap (ap c\_2Eprim\_rec\_2E\_3C (ap (c\_2Efcp\_2Edimindex \\ & A\_27b) (c\_2Ebool\_2Ethet\_value A\_27b))) (ap (c\_2Efcp\_2Edimindex \\ & A\_27a) (c\_2Ebool\_2Ethet\_value A\_27a)))) (ap (c\_2Efcp\_2Edimindex \\ & A\_27b) (c\_2Ebool\_2Ethet\_value A\_27b))) (ap (c\_2Efcp\_2Edimindex \\ & A\_27a) (c\_2Ebool\_2Ethet\_value A\_27a)))) V0v)))) \end{aligned}$$