

thm\_2Ebitstring\_2Eword\_bit\_last\_shiftr  
 (TMUoF-  
 BDt15gY6XwRGisuH8zUoLgCaR22uu7)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (2)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (3)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (4)$$

**Definition 5** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x))$

Let  $c\_2Elist\_2EGENLIST : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2EGENLIST\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{ty\_2Enum\_2Enum})^{(A\_27a^{ty\_2Enum\_2Enum})}) \quad (5)$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (6)$$

**Definition 6** We define  $c\_2Elist\_2EPAD\_LEFT$  to be  $\lambda A\_27a : \iota. \lambda V0c \in A\_27a. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 7** We define  $c\_2Ebitstring\_2Ezero\_extend$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. \lambda V1v \in (ty\_2Elist \_2Elist$

Let  $c\_2Elist\_2ETAKE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETAKE\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}\ ty\_2Enum\_2Enum)) \quad (7)$$

**Definition 8** We define  $c\_2Ebitstring\_2Eshiftr$  to be  $\lambda V0v \in (ty\_2Elist\_2Elist\ 2). \lambda V1m \in ty\_2Enum\_2Enum$

Let  $c\_2Elist\_2EDROP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2EDROP\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}\ ty\_2Enum\_2Enum)) \quad (8)$$

**Definition 9** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t) c\_2Ebool\_2E$

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21\ 2) (\lambda V2t \in$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (11)$$

**Definition 12** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ m)$

**Definition 13** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap\ P\ x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge$  of type  $\iota \Rightarrow \iota$ .

**Definition 14** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a)\ V0P)))$

**Definition 15** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 16** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. (V2t2 \in A\_27a)))$

**Definition 17** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0f \in (A\_27b^{A\_27a}). (\lambda V1x \in A\_27a. (V1x \in A\_27b)))$

**Definition 18** We define  $c\_2Ebitstring\_2Efixwidth$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. \lambda V1v \in (ty\_2Elist\_2Elist\ 2)$

**Definition 19** We define  $c\_2Ebitstring\_2Efield$  to be  $\lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2Enum\_2Enum$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (12)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (13)$$

**Definition 20** We define  $c\_2Ebitstring\_2Etestbit$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1v \in (ty\_2Elist\_2Elist$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (14)$$

Let  $c\_2Ebool\_2Ethethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Ebool\_2Ethethe\_value A\_27a \in (ty\_2Ebool\_2Eitself A\_27a) \quad (15)$$

Let  $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Efcp\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (16)$$

Let  $ty\_2Efcp\_2Efinitelimage : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Efcp\_2Efinitelimage A0) \quad (17)$$

**Definition 21** We define  $c\_2Ebool\_2E_3F_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap c\_2Ebool\_2E_2F_5C$

**Definition 22** We define  $c\_2Efcp\_2Efinitelindex$  to be  $\lambda A\_27a : \iota. (ap (c\_2Emin\_2E_40 (A\_27a^{ty\_2Enum\_2Enum}))$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty\_2Efcp\_2Ecart A0 A1) \quad (18)$$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart A\_27a A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinitelimage A\_27b)})^{(ty\_2Efcp\_2Ecart A\_27a A\_27b)}) \quad (19)$$

**Definition 23** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efcp\_2Ecart A\_27a A\_27b). (ap (ap (c\_2Ebool\_2E_3F_21 A\_27a A\_27b)) (c\_2Efcp\_2Efinitelindex A\_27a A\_27b)))$

**Definition 24** We define  $c\_2Efcp\_2EFCP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (ap (ap (c\_2Ebool\_2E_3F_21 A\_27a A\_27b)) (c\_2Efcp\_2Efinitelindex A\_27a A\_27b)))$

**Definition 25** We define  $c\_2Ebitstring\_2Ev2w$  to be  $\lambda A_27a : \iota. \lambda V0v \in (ty\_2Elist\_2Elist 2).(ap (c\_2Efcp\_2$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (20)$$

**Definition 26** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 27** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (21)$$

**Definition 28** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B$

**Definition 29** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 30** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 31** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(\lambda V2n \in$

**Definition 32** We define  $c\_2Ewords\_2Eword\_bit$  to be  $\lambda A_27a : \iota. \lambda V0b \in ty\_2Enum\_2Enum.\lambda V1w \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}.$

**Definition 33** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A\_27b})^{A\_27c})^{A\_27b})$

**Definition 34** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A_27a : \iota. (ap (ap (c\_2Ecombin\_2ES A_27a (A_27a^{A\_27a}) A_27a)) A_27a))$

Let  $c\_2Elist\_2ENULL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c\_2Elist\_2ENULL A_27a \in (2^{(ty\_2Elist\_2Elist A_27a)}) \quad (22)$$

Let  $c\_2Elist\_2EGENLIST\_AUX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c\_2Elist\_2EGENLIST\_AUX A_27a \in (((ty\_2Elist\_2Elist A_27a)^{(ty\_2Elist\_2Elist A_27a)})^{ty\_2Enum\_2Enum})^{(A_27a^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}} \quad (23)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (24)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (25)$$

**Definition 35** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(\lambda V2n \in$

**Definition 36** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(\lambda V2n \in$

**Definition 37** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2Ebool$

Let  $c\_2Earithmetic\_2EXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (26)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (27)$$

**Definition 38** We define  $c\_2Enumeral\_2EiiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2ESUC (ap$

**Definition 39** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 40** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

Let  $c\_2Elist\_2ELAST : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Elist\_2ELAST A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist A\_27a)}) \quad (28)$$

Let  $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Elist\_2EREVERSE A\_27a \in ((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)}) \quad (29)$$

**Definition 41** We define  $c\_2Erich\_list\_2ELASTN$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1xs \in (ty$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B V1n) V0m)))) \quad (30)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B V1n) V0m)) = (ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p)))) \quad (31)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0m)) V1n))))))) \quad (32)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Enum\_2E0) V0n))) \quad (33)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m)))))) \quad (34)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2E\_2D c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge ((ap (ap c\_2Earithmetic\_2E\_2D V0m) c\_2Enum\_2E0) = V0m))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\ & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\ & (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge \\ & ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\ & (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\ & V1n) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\ & (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\ & V0m) V1n)))))))))) \end{aligned} \quad (36)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\ & ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p))))))) \quad (37)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2D \\ & V0n) V1m)) V0n)))) \quad (38)$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum. (\forall V1c \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2D (ap (ap c\_2Earithmetic\_2E\_2B V0a) \\ & V1c)) V1c) = V0a))) \quad (39)$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2D V0c) \\ & V0c) = c\_2Enum\_2E0)) \quad (40)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p \\
 & \quad ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m)))))) \\
 \end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
 & \quad V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))) \\
 \end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & (\neg(V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
 & \quad V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
 & \quad V1n)) V0m)))))) \\
 \end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap \\
 & c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
 & \quad c\_2Earithmetic\_2EZERO)) V0n))) \\
 \end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2D V0m) \\
 & (ap (ap c\_2Earithmetic\_2E\_2D V1n) V2p)) = (ap (ap (c\_2Ebool\_2ECOND \\
 & ty\_2Enum\_2Enum) (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)) V0m) \\
 & (ap (ap c\_2Earithmetic\_2E\_2D (ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
 & \quad V2p)) V1n)))))) \\
 \end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \forall V2p \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2E\_2D V0m) \\
 & V1n) = V2p) \Leftrightarrow ((V0m = (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) \vee ((p \\
 & (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & V2p) c\_2Enum\_2E0))))))) \\
 \end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0n \in ty\_2Enum\_2Enum. ( \\ & \forall V1v \in (ty\_2Elist\_2Elist 2). ((p (ap (ap (c\_2Ewords\_2Eword\_bit \\ & A_{27a}) V0n) (ap (c\_2Ebitstring\_2Ev2w A_{27a}) V1v))) \Leftrightarrow ((p (ap (ap \\ & c\_2Eprim\_rec\_2E\_3C V0n) (ap (c\_2Efcp\_2Edimindex A_{27a}) (c\_2Ebool\_2Ethe\_value \\ & A_{27a})))) \wedge (p (ap (ap c\_2Ebitstring\_2Etestbit V0n) V1v))))) \\ (47) \end{aligned}$$

Assume the following.

$$True \quad (48)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (50)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (51)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & \forall A_{27b}.nonempty\ A_{27b} \Rightarrow ( \\ & \forall V0f \in (A_{27b}^{A_{27a}}). (\forall V1x \in A_{27a}. ((ap (ap (c\_2Ebool\_2ELET \\ & A_{27a} A_{27b}) V0f) V1x) = (ap V0f V1x)))) \quad (52) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0t \in 2. ((\forall V1x \in \\ & A_{27a}. (p V0t) \Leftrightarrow (p V0t))) \quad (53) \end{aligned}$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (54) \end{aligned}$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \quad (55) \end{aligned}$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (56) \end{aligned}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (57)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (58)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (59)$$

Assume the following.

$$((\neg(True \Leftrightarrow False)) \wedge (\neg(False \Leftrightarrow True))) \quad (60)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (61)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0t1 \in A\_27a.(\forall V1t2 \in \\ & A\_27a.(((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) \\ & V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (62)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B)))))))))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (65)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (66)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (67)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) V1Q) V3x\_27) \\ & V5y\_27)))))))))) \end{aligned} \quad (69)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1a \in \\ & A\_27a. ((\exists V2x \in A\_27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p ( \\ & ap V0P V1a)))))) \end{aligned} \quad (70)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \\ & (\forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap (ap (c\_2Ecombin\_2EK \\ & A\_27a A\_27b) V0x) V1y) = V0x))) \end{aligned} \quad (71)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap (c\_2Ecombin\_2EI \\ & A\_27a) V0x) = V0x)) \quad (72)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (((p (ap (c\_2Elist\_2ENULL A\_27a) \\ & (c\_2Elist\_2ENIL A\_27a))) \Leftrightarrow \text{True}) \wedge (\forall V0h \in A\_27a. (\forall V1t \in \\ & (ty\_2Elist\_2Elist A\_27a). ((p (ap (c\_2Elist\_2ENULL A\_27a) (ap \\ & (ap (c\_2Elist\_2ECONS A\_27a) V0h) V1t))) \Leftrightarrow \text{False}))) \end{aligned} \quad (73)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist \\ & A\_27a). ((ap (ap (c\_2Elist\_2EAPPEND A\_27a) (c\_2Elist\_2ENIL A\_27a)) \\ & V0l) = V0l)) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist A\_27a). (\forall V2l2 \in \\ & (ty\_2Elist\_2Elist A\_27a). (\forall V3h \in A\_27a. ((ap (ap (c\_2Elist\_2EAPPEND \\ & A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V3h) V1l1)) V2l2) = (ap (ap \\ & (c\_2Elist\_2ECONS A\_27a) V3h) (ap (ap (c\_2Elist\_2EAPPEND A\_27a) \\ & V1l1) V2l2))))))) \end{aligned} \quad (74)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (((ap\ (c\_2Elist\_2ELENGTH\ A_{27a}) \\ & (c\_2Elist\_2ENIL\ A_{27a})) = c\_2Enum\_2E0) \wedge (\forall V0h \in A_{27a}.( \\ & \forall V1t \in (ty\_2Elist\_2Elist\ A_{27a}).((ap\ (c\_2Elist\_2ELENGTH\ \\ & A_{27a})\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A_{27a})\ V0h)\ V1t)) = (ap\ c\_2Enum\_2ESUC \\ & (ap\ (c\_2Elist\_2ELENGTH\ A_{27a})\ V1t))))))) \end{aligned} \quad (75)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0a0 \in A_{27a}.(\forall V1a1 \in \\ & (ty\_2Elist\_2Elist\ A_{27a}).(\forall V2a0\_27 \in A_{27a}.(\forall V3a1\_27 \in \\ & (ty\_2Elist\_2Elist\ A_{27a}).(((ap\ (ap\ (c\_2Elist\_2ECONS\ A_{27a})\ V0a0) \\ & V1a1) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A_{27a})\ V2a0\_27)\ V3a1\_27)) \Leftrightarrow ((V0a0 = \\ & V2a0\_27) \wedge (V1a1 = V3a1\_27))))))) \end{aligned} \quad (76)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0a1 \in (ty\_2Elist\_2Elist\ \\ & A_{27a}).(\forall V1a0 \in A_{27a}.(\neg((c\_2Elist\_2ENIL\ A_{27a}) = (ap\ ( \\ & ap\ (c\_2Elist\_2ECONS\ A_{27a})\ V1a0)\ V0a1)))))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0l \in (ty\_2Elist\_2Elist\ \\ & A_{27a}).(((ap\ (c\_2Elist\_2ELENGTH\ A_{27a})\ V0l) = c\_2Enum\_2E0) \Leftrightarrow \\ & (V0l = (c\_2Elist\_2ENIL\ A_{27a})))))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0l \in (ty\_2Elist\_2Elist\ \\ & A_{27a}).((p\ (ap\ (c\_2Elist\_2ENULL\ A_{27a})\ V0l)) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ \\ & A_{27a})))))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0l \in (ty\_2Elist\_2Elist\ \\ & A_{27a}).((p\ (ap\ (c\_2Elist\_2ENULL\ A_{27a})\ V0l)) \Leftrightarrow ((ap\ (c\_2Elist\_2ELENGTH\ \\ & A_{27a})\ V0l) = c\_2Enum\_2E0)))) \end{aligned} \quad (80)$$

Assume the following.

Assume the following.

$$\begin{aligned} \forall A.27a.\text{nonempty } A.27a \Rightarrow & ((\forall V0x \in A.27a.((ap(c.2Elist.2ELAST \\ A.27a) (ap(ap(c.2Elist.2ECONS A.27a) V0x) (c.2Elist.2ENIL A.27a))) = \\ V0x)) \wedge (\forall V1x \in A.27a.(\forall V2y \in A.27a.(\forall V3z \in ( \\ ty.2Elist.2Elist A.27a).(ap(c.2Elist.2ELAST A.27a) (ap(ap \\ (c.2Elist.2ECONS A.27a) V1x) (ap(ap(c.2Elist.2ECONS A.27a) V2y) \\ V3z))) = (ap(c.2Elist.2ELAST A.27a) (ap(ap(c.2Elist.2ECONS A.27a) \\ V2y) V3z))))))) \\ (82) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\forall V0f \in (A_{27a}^{ty\_2Enum\_2Enum}). \\
& (\forall V1l \in (ty\_2Elist\_2Elist A_{27a}).((ap (ap (ap (c\_2Elist\_2EGENLIST\_AUX \\
& A_{27a}) V0f) c\_2Enum\_2E0) V1l) = V1l))) \wedge ((\forall V2f \in (A_{27a}^{ty\_2Enum\_2Enum}. \\
& (\forall V3n \in ty\_2Enum\_2Enum.(\forall V4l \in (ty\_2Elist\_2Elist \\
& A_{27a}).((ap (ap (ap (c\_2Elist\_2EGENLIST\_AUX A_{27a}) V2f) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 V3n))) V4l) = (ap (ap (ap (c\_2Elist\_2EGENLIST\_AUX \\
& A_{27a}) V2f) (ap (ap c\_2Earithmetic\_2E\_2D (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 V3n))) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) (ap (ap \\
& (c\_2Elist\_2ECONS A_{27a}) (ap V2f (ap (ap c\_2Earithmetic\_2E\_2D ( \\
& ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V3n))) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \\
& V4l)))))) \wedge (\forall V5f \in (A_{27a}^{ty\_2Enum\_2Enum}).(\forall V6n \in \\
& ty\_2Enum\_2Enum.(\forall V7l \in (ty\_2Elist\_2Elist A_{27a}).((ap \\
& (ap (ap (c\_2Elist\_2EGENLIST\_AUX A_{27a}) V5f) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 V6n))) V7l) = (ap (ap (ap (c\_2Elist\_2EGENLIST\_AUX \\
& A_{27a}) V5f) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& V6n))) (ap (ap (c\_2Elist\_2ECONS A_{27a}) (ap V5f (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 V6n)))) V7l))))))) \\
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0f \in (A_{27a}^{ty\_2Enum\_2Enum}). \\
& (\forall V1n \in ty\_2Enum\_2Enum.((ap (ap (c\_2Elist\_2EGENLIST A_{27a}) \\
& V0f) c\_2Enum\_2E0) = (c\_2Elist\_2ENIL A_{27a})) \wedge ((ap (ap (c\_2Elist\_2EGENLIST \\
& A_{27a}) V0f) (ap c\_2Earithmetic\_2ENUMERAL V1n)) = (ap (ap (ap (c\_2Elist\_2EGENLIST\_AUX \\
& A_{27a}) V0f) (ap c\_2Earithmetic\_2ENUMERAL V1n)) (c\_2Elist\_2ENIL \\
& A_{27a})))))) \\
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& (((ap c\_2Enum\_2ESUC c\_2Earithmetic\_2EZERO) = (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO)) \wedge ((\forall V0n \in ty\_2Enum\_2Enum.((ap \\
& c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2EBIT1 V0n)) = (ap c\_2Earithmetic\_2EBIT2 \\
& V0n))) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2EBIT2 \\
& V1n)) = (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Enum\_2ESUC V1n))))))) \\
\end{aligned} \tag{85}$$

Assume the following.

$(\forall V0n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2B c_2Enum\_2E0) V0n) = V0n)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2B V1n) c_2Enum\_2E0) = V1n)) \wedge (\forall V2n \in ty\_2Enum\_2Enum. (\forall V3m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2B V3m) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Enum\_2EiZ (ap (ap c_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge (\forall V4n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2A c_2Enum\_2E0) V4n) = c_2Enum\_2E0)) \wedge (\forall V5n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2A V5n) c_2Enum\_2E0) = c_2Enum\_2E0)) \wedge (\forall V6n \in ty\_2Enum\_2Enum. (\forall V7m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2A (ap c_2Earithmetic\_2ENUMERAL V6n)) (ap c_2Earithmetic\_2ENUMERAL V7m)) = (ap c_2Earithmetic\_2ENUMERAL (ap (ap c_2Earithmetic\_2E\_2A V6n) V7m)))))) \wedge (\forall V8n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2D c_2Enum\_2E0) V8n) = c_2Enum\_2E0)) \wedge (\forall V9n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2D V9n) c_2Enum\_2E0) = V9n)) \wedge (\forall V10n \in ty\_2Enum\_2Enum. (\forall V11m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2D (ap c_2Earithmetic\_2ENUMERAL V10n)) (ap c_2Earithmetic\_2ENUMERAL V11m)) = (ap c_2Earithmetic\_2ENUMERAL (ap (ap c_2Earithmetic\_2E\_2D V10n) V11m)))))) \wedge (\forall V12n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT1 V12n)))) = c_2Enum\_2E0)) \wedge (\forall V13n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT2 V13n)))) = c_2Enum\_2E0)) \wedge (\forall V14n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP V14n) c_2Enum\_2E0) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT1 c_2Earithmetic\_2EZERO)))))) \wedge (\forall V15n \in ty\_2Enum\_2Enum. (\forall V16m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP (ap c_2Earithmetic\_2ENUMERAL V15n)) (ap c_2Earithmetic\_2ENUMERAL V16m)) = (ap c_2Earithmetic\_2ENUMERAL (ap (ap c_2Earithmetic\_2EEEXP V15n) V16m)))))) \wedge (((ap c_2Enum\_2ESUC c_2Enum\_2E0) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT1 c_2Earithmetic\_2EZERO)))) \wedge (\forall V17n \in ty\_2Enum\_2Enum. ((ap c_2Enum\_2ESUC (ap c_2Earithmetic\_2ENUMERAL V17n)) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Enum\_2ESUC V17n)))))) \wedge (((ap c_2Eprim\_rec\_2EPRE c_2Enum\_2E0) = c_2Enum\_2E0) \wedge (\forall V18n \in ty\_2Enum\_2Enum. ((ap c_2Eprim\_rec\_2EPRE (ap c_2Earithmetic\_2ENUMERAL V18n)) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Eprim\_rec\_2EPRE V18n)))))) \wedge (\forall V19n \in ty\_2Enum\_2Enum. (((ap c_2Earithmetic\_2ENUMERAL V19n) = c_2Enum\_2E0) \Leftrightarrow (V19n = c_2Earithmetic\_2EZERO))) \wedge (\forall V20n \in ty\_2Enum\_2Enum. ((c_2Enum\_2E0 = (ap c_2Earithmetic\_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic\_2EZERO))) \wedge (\forall V21n \in ty\_2Enum\_2Enum. ((\forall V22m \in ty\_2Enum\_2Enum. (((ap c_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge ((\forall V23n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim\_rec\_2E\_3C V23n) c_2Enum\_2E0)) \Leftrightarrow False))) \wedge (\forall V24n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim\_rec\_2E\_3C c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL V24n))) \Leftrightarrow (p (ap (ap c_2Eprim\_rec\_2E\_3C c_2Earithmetic\_2EZERO) V24n)))))) \wedge (\forall V25n \in ty\_2Enum\_2Enum. (\forall V26m \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim\_rec\_2E\_3C (ap c_2Earithmetic\_2ENUMERAL V25n)) (ap c_2Earithmetic\_2ENUMERAL V26m)))) \Leftrightarrow (p (ap (ap c_2Eprim\_rec\_2E\_3C V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V27n)) \Leftrightarrow False))) \wedge (\forall V28n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E (ap c_2Enum\_2E0) V28n)) \Leftrightarrow (p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V28n)))) \wedge (\forall V29n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E (ap c_2Earithmetic\_2ENUMERAL V29n)) \Leftrightarrow (p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V29n)))) \wedge (\forall V30m \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V30m)) \Leftrightarrow (p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V30m)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3D c_2Enum\_2E0) V31n)) \Leftrightarrow True))) \wedge (\forall V32n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3D c_2Enum\_2E0) V32n)) \Leftrightarrow False))) \wedge ((p (ap (ap c_2Earithmetic\_2E\_3D c_2Enum\_2E0) V32n)) \Leftrightarrow True)))$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& (ap c_2Earithmetic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg(p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))) \\
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n))) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\
& A\_27a). ((\neg(V0l = (c\_2Elist\_2ENIL A\_27a))) \Rightarrow ((ap (ap (c\_2Erich\_list\_2ELASTN \\
& A\_27a) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))) V0l) = (ap (ap (c\_2Elist\_2ECONS A\_27a) \\
& (ap (c\_2Elist\_2ELAST A\_27a) V0l)) (c\_2Elist\_2ENIL A\_27a)))))) \\
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0n \in ty\_2Enum\_2Enum. ( \\ \forall V1l \in (ty\_2Elist\_2Elist\ A_{27a}).((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\ A_{27a})\ (ap\ (c\_2Elist\_2ELLENGTH\ A_{27a})\ V1l))) \Rightarrow ((ap\ (ap\ (c\_2Elist\_2EDROP \\ A_{27a})\ V0n)\ V1l) = (ap\ (ap\ (c\_2Erich\_list\_2ELASTN\ A_{27a})\ (ap\ (ap \\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Elist\_2ELLENGTH\ A_{27a})\ V1l))\ V0n)) \\ V1l)))))) \end{aligned} \quad (91)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (92)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (93)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))) \quad (94)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (95)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (96)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow \\ (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow ((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p \\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \end{aligned} \quad (97)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow \\ (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow ((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{aligned} \quad (98)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow \\ (p\ V1q) \vee (p\ V2r))) \Leftrightarrow ((p\ V0p) \vee ((\neg(p\ V1q)) \wedge ((p\ V0p) \vee (\neg(p\ V2r)))) \wedge \\ ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{aligned} \quad (99)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee \\ (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \quad (100)$$

**Theorem 1**

$$\begin{aligned}
 & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0i \in ty\_2Enum\_2Enum. \\
 & \quad \forall V1v \in (ty\_2Elist\_2Elist 2).((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
 & \quad \quad V0i) (ap (c\_2Efcp\_2Edimindex A\_27a) (c\_2Ebool\_2Ethe\_value A\_27a)))))) \Rightarrow \\
 & \quad ((p (ap (ap (c\_2Ewords\_2Eword\_bit A\_27a) V0i) (ap (c\_2Ebitstring\_2Ev2w \\
 & \quad \quad A\_27a) V1v))) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2ELET (ty\_2Elist\_2Elist 2) \\
 & \quad \quad 2) (\lambda V2l \in (ty\_2Elist\_2Elist 2).(ap (ap c\_2Ebool\_2E\_2F\_5C \\
 & \quad \quad (ap c\_2Ebool\_2E\_7E (ap (c\_2Elist\_2ENULL 2) V2l))) (ap (c\_2Elist\_2ELAST \\
 & \quad \quad 2) V2l)))) (ap (ap c\_2Ebitstring\_2Eshiftr V1v) V0i)))))))
 \end{aligned}$$