

thm_2Ebitstring_2Eword__extract__v2w (TM- RAn5xPbkqsRZG2enYAhEy4HMubBWQadSD)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ENIL\ A.27a \in (ty_2Elist_2Elist\ A.27a) \quad (2)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ECONS\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})^{A.27a}) \quad (3)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (4)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ELENGTH\ A.27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A.27a)}) \quad (5)$$

Let $c_2Earithmetic_2E2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Elist_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ETAKE\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 13 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A) \text{ of type } \iota \Rightarrow \iota$.

Definition 14 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40$

Definition 15 We define `c_2Eprim_rec_2E_3C` to be $\lambda V0m \in \text{ty_2Enum_2Enum}. \lambda V1n \in \text{ty_2Enum_2Enum}$

Definition 16 We define `c_2Ebool_2ECOND` to be $\lambda A. 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 27a. (\lambda V2t2 \in A. 27a. ($

Definition 17 We define `c_2Ebool_2ELET` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. (\lambda V0f \in (A. 27b^{A-27a}). (\lambda V1x \in A. 27$

Definition 18 We define `c_2Ebitstring_2Efixwidth` to be $\lambda V0n \in \text{ty_2Enum_2Enum}. \lambda V1v \in (\text{ty_2Elist_2E}$

Definition 19 We define `c_2Ebitstring_2Efield` to be $\lambda V0h \in \text{ty_2Enum_2Enum}. \lambda V1l \in \text{ty_2Enum_2Enum}$

Definition 20 We define `c_2Ebitstring_2Etestbit` to be $\lambda V0b \in \text{ty_2Enum_2Enum}. \lambda V1v \in (\text{ty_2Elist_2E}$

Let `ty_2Efcf_2Efinite_image` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty (ty_2Efcf_2Efinite_image } A0) \quad (14)$$

Let `ty_2Ebool_2Eitself` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty (ty_2Ebool_2Eitself } A0) \quad (15)$$

Let `c_2Ebool_2Ethe_value` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Ebool_2Ethe_value } A. 27a \in (\text{ty_2Ebool_2Eitself } A. 27a) \quad (16)$$

Let `c_2Efcf_2Edimindex` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Efcf_2Edimindex } A. 27a \in (\text{ty_2Enum_2Enum}^{(\text{ty_2Ebool_2Eitself } A. 27a)}) \quad (17)$$

Definition 21 We define `c_2Ebool_2E_3F_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap (ap c_2Ebool_2E_2F_5C$

Definition 22 We define `c_2Efcf_2Efinite_index` to be $\lambda A. 27a : \iota. (\text{ap (c_2Emin_2E_40 (A_27a}^{\text{ty_2Enum_2Enum}}$

Let `ty_2Efcf_2Ecart` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty (ty_2Efcf_2Ecart } A0 \ A1) \quad (18)$$

Let `c_2Efcf_2Edest_cart` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Efcf_2Edest_cart } A. 27a \ A. 27b \in ((A_27a^{(\text{ty_2Efcf_2Efinite_image } A. 27b)})^{\text{ty_2Efcf_2Ecart } A. 27a \ A. 27b}) \quad (19)$$

Definition 23 We define `c_2Efcf_2Efcf_index` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0x \in (\text{ty_2Efcf_2Ecart } A. 27$

Definition 24 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 25 We define $c_2Ebitstring_2Ev2w$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Elist_2Elist\ 2).(ap (c_2Efc_2E$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (21)$$

Definition 26 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 27 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 28 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap\ c_2Earithmetic$

Definition 29 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 30 We define $c_2Earithmetic_2EMIN$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 31 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 32 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 33 We define $c_2Ewords_2Eword_bits$ to be $\lambda A_27a : \iota.\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum$

Definition 34 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap\ c_2Earithmetic$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (22)$$

Definition 35 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (23)$$

Definition 36 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap (ap$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (24)$$

Definition 37 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (25)$$

Definition 38 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 39 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 40 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 41 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcpc_2EFC$

Definition 42 We define $c_2Ewords_2Ew2w$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w \in (ty_2Efcpc_2Ecart\ 2\ A_27a$

Definition 43 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Definition 44 We define $c_2Ewords_2Eword_extract$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0h \in ty_2Enum_2Enum$

Assume the following.

$$True \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}). \\ & (\forall V2x \in A_27c.((ap (ap (ap (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a) \\ & V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \end{aligned} \quad (28)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0h \in ty_2Enum_2Enum.(\forall V1l \in ty_2Enum_2Enum.(\forall V2v \in \\ & (ty_2Elist_2Elist\ 2).((ap (ap (ap (c_2Ewords_2Eword_extract \\ & A_27a\ A_27b)\ V0h)\ V1l)\ (ap (c_2Ebitstring_2Ev2w\ A_27a)\ V2v)) = (\\ & ap (c_2Ewords_2Ew2w\ A_27a\ A_27b)\ (ap (ap (ap (c_2Ewords_2Eword_bits \\ & A_27a)\ V0h)\ V1l)\ (ap (c_2Ebitstring_2Ev2w\ A_27a)\ V2v)))))) \end{aligned}$$