

thm\_2Ebitstring\_2Eword\_\_join\_\_v2w\_\_rwt  
(TMSVV2HYr5m3X5kAJahLUBkzbuEfqtxiukr)

October 26, 2020

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (1)$$

Let  $c\_2Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (2)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (3)$$

Let  $c\_2EfcP\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2EfcP\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (4)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (5)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (6)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Elist\_2EDROP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EDROP\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_Ebool\_2ET$  to be  $(ap (ap (c\_Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_Ebool\_2EF$  to be  $(ap (c\_Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 5** We define  $c\_Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Let  $c\_Elist\_2EGENLIST : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Elist\_2EGENLIST A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{ty\_2Enum\_2Enum})^{(A\_27a^{ty\_2Enum\_2Enum})}) \quad (9)$$

Let  $c\_Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Elist\_2EAPPEND A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{(ty\_2Elist\_2Elist A\_27a)}) \quad (10)$$

**Definition 6** We define  $c\_Elist\_2EPAD\_LEFT$  to be  $\lambda A\_27a : \iota.\lambda V0c \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 7** We define  $c\_Ebitstring\_2Ezero\_extend$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1v \in (ty\_2Elist\_2Elist$

**Definition 8** We define  $c\_Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_2E\_3D\_3D\_3E V0t) c\_Ebool\_2EF$

**Definition 10** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21 2)) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (12)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (13)$$

**Definition 11** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

**Definition 12** We define  $c\_Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_Emin\_2E\_40$

**Definition 14** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 15** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 16** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0f \in (A\_27b^{A\_27a}). (\lambda V1x \in A\_27$

**Definition 17** We define  $c\_2Ebitstring\_2Efixwidth$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. \lambda V1v \in (ty\_2Elist\_2E$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (14)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (15)$$

Let  $c\_2Elist\_2ETAKE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETAKE\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (16)$$

**Definition 18** We define  $c\_2Ebitstring\_2Eshiftr$  to be  $\lambda V0v \in (ty\_2Elist\_2Elist\ 2). \lambda V1m \in ty\_2Enum\_2Enum$

**Definition 19** We define  $c\_2Ebitstring\_2Efield$  to be  $\lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2Enum\_2Enum$

**Definition 20** We define  $c\_2Ebitstring\_2Etestbit$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1v \in (ty\_2Elist\_2Elist$

Let  $ty\_2Efcf\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcf\_2Efinite\_image\ A0) \quad (17)$$

**Definition 21** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ c\_2Ebool\_2E\_2F\_25C$

**Definition 22** We define  $c\_2Efcf\_2Efinite\_index$  to be  $\lambda A\_27a : \iota. (ap\ (c\_2Emin\_2E\_40\ (A\_27a^{ty\_2Enum\_2Enum}$

Let  $ty\_2Efcf\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcf\_2Ecart\ A0\ A1) \quad (18)$$

Let  $c\_2Efcf\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Efcf\_2Edest\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcf\_2Efinite\_image\ A\_27b)})^{(ty\_2Efcf\_2Ecart\ A\_27a\ A\_27b)}) \quad (19)$$

**Definition 23** We define  $c\_2Efcf\_2Efcf\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efcf\_2Ecart\ A\_27a$

**Definition 24** We define  $c\_2Efcf\_2EFCF$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (ap$

**Definition 25** We define  $c\_2Ebitstring\_2Ev2w$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Elist\_2Elist\ 2).(ap\ (c\_2Efcpr\_2Efcpr\ 2)\ A\_27a)$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (20)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (21)$$

**Definition 26** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 27** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (22)$$

**Definition 28** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n))$

**Definition 29** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (23)$$

**Definition 30** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ c\_2Ebit\_2Ebit\ V0b)\ V1n))$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (24)$$

**Definition 31** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcpr\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c\_2Ewords\_2Ewords\ V0w))$

**Definition 32** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n))$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (25)$$

**Definition 33** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V0x$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (26)$$

**Definition 34** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 35** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 36** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

**Definition 37** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap (c\_2Efc$

**Definition 38** We define  $c\_2Ewords\_2Ew2w$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w \in (ty\_2Efc$

**Definition 39** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21$

**Definition 40** We define  $c\_2Earithmetic\_2E\_3C\_2D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

**Definition 41** We define  $c\_2Ewords\_2Eword\_2Isl$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc$

**Definition 42** We define  $c\_2Ewords\_2Eword\_2or$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efc$

**Definition 43** We define  $c\_2Ewords\_2Eword\_2join$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in (ty\_2Efc$

**Definition 44** We define  $c\_2Epred\_2set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2ET)$ .

**Definition 45** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A-27a}).(ap$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (27)$$

Let  $c\_2Epair\_2EABS\_2prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_2prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A-27b})^{A-27a}}) \quad (28)$$

**Definition 46** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2$

Let  $c\_2Epred\_2set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_2set\_2EGSPEC A\_27a A\_27b \in ((2^{A-27a})^{((ty\_2Epair\_2Eprod A\_27a 2)^{A-27b})}) \quad (29)$$

**Definition 47** We define  $c\_2Epred\_2set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A-27a}).(ap (c\_2$

**Definition 48** We define  $c\_2Epred\_2set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 49** We define  $c\_2Epred\_2set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c\_2Ebool\_2E\_21$

**Definition 50** We define  $c\_2Ecombin\_2EFAIL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0v1 \in (ty\_2Elist\_2Elist\ 2).(\forall V1v2 \in (ty\_2Elist\_2Elist \\ & \quad 2).(((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ (c\_2Epred\_set\_2EUNIV \\ & \quad A\_27a)))) \wedge (p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27b)\ (c\_2Epred\_set\_2EUNIV \\ & \quad A\_27b)))) \Rightarrow ((ap\ (ap\ (c\_2Ewords\_2Eword\_join\ A\_27a\ A\_27b)\ (ap\ ( \\ & \quad c\_2Ebitstring\_2Ev2w\ A\_27a)\ V0v1))\ (ap\ (c\_2Ebitstring\_2Ev2w\ A\_27b) \\ & \quad V1v2)) = (ap\ (c\_2Ebitstring\_2Ev2w\ (ty\_2Esum\_2Esum\ A\_27a\ A\_27b)) \\ & \quad (ap\ (ap\ (c\_2Elist\_2EAPPEND\ 2)\ V0v1)\ (ap\ (ap\ c\_2Ebitstring\_2Efixwidth \\ & \quad (ap\ (c\_2Efcg\_2Edimindex\ A\_27b)\ (c\_2Ebool\_2Ethe\_value\ A\_27b)) \\ & \quad V1v2))))))))) \end{aligned} \tag{30}$$

Assume the following.

$$True \tag{31}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{32}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \tag{34}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & \quad (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & \quad (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{37}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & \quad (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & \quad p\ V0t)))))) \end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\ V5y\_27)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap\ (ap\ (c\_2Ecombin\_2EFAIL \\ A\_27a\ A\_27b)\ V0x)\ V1y) = V0x))) \end{aligned} \quad (42)$$

### Theorem 1

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0bad\_20domain \in 2. (\forall V1v1 \in (ty\_2Elist\_2Elist \\ 2). (\forall V2v2 \in (ty\_2Elist\_2Elist\ 2). ((ap\ (ap\ (c\_2Ewords\_2Eword\_join \\ A\_27a\ A\_27b)\ (ap\ (c\_2Ebitstring\_2Ev2w\ A\_27a)\ V1v1))\ (ap\ (c\_2Ebitstring\_2Ev2w \\ A\_27b)\ V2v2))) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (ty\_2Efcpcart\ 2 \\ (ty\_2Esum\_2Esum\ A\_27a\ A\_27b)))\ (ap\ (ap\ c\_2Ebool\_2E\_2F\_5C\ (ap\ ( \\ c\_2Epred\_set\_2EFINITE\ A\_27a)\ (c\_2Epred\_set\_2EUNIV\ A\_27a)))) \\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27b)\ (c\_2Epred\_set\_2EUNIV\ A\_27b)))) \\ (ap\ (c\_2Ebitstring\_2Ev2w\ (ty\_2Esum\_2Esum\ A\_27a\ A\_27b))\ (ap\ (ap \\ (c\_2Elist\_2EAPPEND\ 2)\ V1v1)\ (ap\ (ap\ c\_2Ebitstring\_2Efixwidth \\ (ap\ (c\_2Efcpcardindex\ A\_27b)\ (c\_2Ebool\_2Ethe\_value\ A\_27b)) \\ V2v2))))\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecombin\_2EFAIL\ (((ty\_2Efcpcart\ 2 \\ A\_27b)\ (ty\_2Efcpcart\ 2\ A\_27a) \\ 2\ (ty\_2Esum\_2Esum\ A\_27a\ A\_27b))\ (ty\_2Efcpcart\ 2\ A\_27b))\ (ty\_2Efcpcart\ 2\ A\_27a) \\ 2) \\ (c\_2Ewords\_2Eword\_join\ A\_27a\ A\_27b))\ V0bad\_20domain)\ (ap \\ (c\_2Ebitstring\_2Ev2w\ A\_27a)\ V1v1))\ (ap\ (c\_2Ebitstring\_2Ev2w \\ A\_27b)\ V2v2)))))) \end{aligned}$$