

thm_2Ebitstring_2Eword_lsb_v2w
(TMcZFdrvaoYdZJDFikHEjuXdSHEYhYPtCZz)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2E_2D) n)$

Definition 8 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (8)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (9)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (10)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELENGTH A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist A_27a)}) \quad (11)$$

Let $c_2Elist_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ETAKE A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{ty_2Enum_2Enum}) \quad (12)$$

Definition 9 We define $c_2Ebitstring_2Eshiftr$ to be $\lambda V0v \in (ty_2Elist_2Elist 2).\lambda V1m \in ty_2Enum_2Enum$

Let $c_2Elist_2EDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EDROP A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 10 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 11 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Let $c_2Elist_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EGENLIST A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Enum_2Enum)}_{(A_27a^{ty_2Enum_2Enum})}) \quad (14)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (15)$$

Definition 12 We define $c_Elist_2EPAD_LEFT$ to be $\lambda A_27a : \iota. \lambda V0c \in A_27a. \lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_Ebitstring_2Ezero_extend$ to be $\lambda V0n \in ty_2Enum_2Enum. \lambda V1v \in (ty_2Elist$

Definition 14 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 15 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E$

Definition 16 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Definition 17 We define $c_Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge$ of type $\iota \Rightarrow \iota$.

Definition 18 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_Emin_2E_40$

Definition 19 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 20 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 21 We define c_Ebool_2ELET to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A_27a}). (\lambda V1x \in A_27b$

Definition 22 We define $c_Ebitstring_2Efixwidth$ to be $\lambda V0n \in ty_2Enum_2Enum. \lambda V1v \in (ty_2Elist_2E$

Definition 23 We define $c_Ebitstring_2Efield$ to be $\lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum$

Definition 24 We define $c_Ebitstring_2Etestbit$ to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1v \in (ty_2Elist_2E$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Efc_2Efinite_image A0) \quad (16)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (17)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (18)$$

Let $c_2Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Efc_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (19)$$

Definition 25 We define $c_Ebool_2E_3F_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap c_Ebool_2E_2F_5C$

Definition 26 We define $c_2Efc_2Efinite_index$ to be $\lambda A_27a : \iota. (ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum$

Let $ty_2Efc_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efc_2Ecart\ A0\ A1) \quad (20)$$

Let $c_2Efc_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efc_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efc_2Efinite_image\ A_27b)})^{(ty_2Efc_2Ecart\ A_27a\ A_27b)}) \quad (21)$$

Definition 27 We define $c_2Efc_2Efc_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efc_2Ecart\ A_27a\ A_27b)$

Definition 28 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap\ (c_2Efc_2Efc_index\ A_27a\ A_27b)\ g))$

Definition 29 We define $c_2Ebitstring_2Ev2w$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Elist_2Elist\ 2).(ap\ (c_2Efc_2Efc_index\ A_27a\ A_27a)\ v)$

Definition 30 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ t1)\ t2))$

Definition 31 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 32 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ A_27a)\ A_27a)\ A_27a)$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EHD\ A_27a \in (A_27a^{(ty_2Elist_2Elist\ A_27a)}) \quad (22)$$

Let $c_2Elist_2ENULL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENULL\ A_27a \in (2^{(ty_2Elist_2Elist\ A_27a)}) \quad (23)$$

Let $c_2Elist_2ELAST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELAST\ A_27a \in (A_27a^{(ty_2Elist_2Elist\ A_27a)}) \quad (24)$$

Definition 33 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2E_21\ 2)\ m)\ V0)\ V0)$

Let $c_2Elist_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEL\ A_27a \in ((A_27a^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \quad (25)$$

Definition 34 We define $c_2Ewords_2Eword_lsb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap\ (c_2Ebitstring_2Ev2w\ A_27a)\ w)$

Definition 35 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap\ (c_2Ebitstring_2Ev2w\ m)\ n)$

Definition 36 We define $c_2Ewords_2Eword_bit$ to be $\lambda A_27a : \iota.\lambda V0b \in ty_2Enum_2Enum.\lambda V1w \in (ty_2Elist_2Elist\ 2).(ap\ (c_2Ebitstring_2Ev2w\ A_27a)\ w)$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D V0m) c_2Enum_2E0) = V0m)))) \quad (26)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE V0m) = (ap (ap c_2Earithmetic_2E_2D V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \quad (27)$$

Assume the following.

$$(\forall V0b \in ty_2Enum_2Enum.(\forall V1v \in (ty_2Elist_2Elist 2).((p (ap (ap c_2Ebitstring_2Etestbit V0b) V1v)) \Leftrightarrow (p (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum 2) (\lambda V2n \in ty_2Enum_2Enum. (ap (ap c_2Ebool_2E_2F_5C (ap (ap c_2Eprim_rec_2E_3C V0b) V2n)) (ap (ap (c_2Elist_2EEL 2) (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2D V2n) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) V0b) V1v)))))) (ap (c_2Elist_2ELENGTH 2) V1v)))))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\forall V1v \in (ty_2Elist_2Elist 2).((p (ap (ap c_2Ewords_2Eword_bit A_27a) V0n) (ap (c_2Ebitstring_2Ev2w A_27a) V1v))) \Leftrightarrow ((p (ap c_2Eprim_rec_2E_3C V0n) (ap (c_2EfcP_2Edimindex A_27a) (c_2Ebool_2Ethe_value A_27a)))) \wedge (p (ap (ap c_2Ebitstring_2Etestbit V0n) V1v)))))) \quad (29)$$

Assume the following.

$$True \quad (30)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg (p V0t)))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1x \in A_27a.((ap (ap c_2Ebool_2ELET A_27a A_27b) V0f) V1x) = (ap V0f V1x)))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (35)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Ecombin_2El A_27a) V0x) = V0x)) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (((ap (c_2Elist_2ELENGTH A_27a) (c_2Elist_2ENIL A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a.(\forall V1t \in (ty_2Elist_2Elist A_27a).((ap (c_2Elist_2ELENGTH A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V0h) V1t)) = (ap c_2Enum_2ESUC (ap (c_2Elist_2ELENGTH A_27a) V1t)))))) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist A_27a).((p (ap (c_2Elist_2ENULL A_27a) V0l)) \Leftrightarrow (V0l = (c_2Elist_2ENIL A_27a)))) \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\forall V1l \in A_27b.(\forall V2ls \in (ty_2Elist_2Elist A_27b).(((ap (c_2Elist_2EEL A_27a) c_2Enum_2E0) = (c_2Elist_2EHD A_27a)) \wedge ((ap (ap (c_2Elist_2EEL A_27b) (ap c_2Enum_2ESUC V0n)) (ap (ap (c_2Elist_2ECONS A_27b) V1l) V2ls)) = (ap (ap (c_2Elist_2EEL A_27b) V0n) V2ls)))))) \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ A_27a).((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0)\ (ap\ (c_2Elist_2ELENGTH \\ A_27a)\ V0l))) \Leftrightarrow (\neg(p\ (ap\ (c_2Elist_2ENULL\ A_27a)\ V0l)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ A_27a).((\neg(V0l = (c_2Elist_2ENIL\ A_27a))) \Rightarrow ((ap\ (ap\ (c_2Elist_2EEL \\ A_27a)\ (ap\ c_2Eprim_rec_2EPRE\ (ap\ (c_2Elist_2ELENGTH\ A_27a) \\ V0l)))\ V0l) = (ap\ (c_2Elist_2ELAST\ A_27a)\ V0l)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0) \\ (ap\ (c_2Efcf_2Edimindex\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Efcf_2Ecart \\ 2\ A_27a).(\forall V1b \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ V1b)\ (ap\ (c_2Efcf_2Edimindex\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)))))) \Rightarrow \\ ((p\ (ap\ (ap\ (c_2Efcf_2Efcf_index\ 2\ A_27a)\ V0w)\ V1b)) \Leftrightarrow (p\ (ap\ (ap \\ (c_2Ewords_2Eword_bit\ A_27a)\ V1b)\ V0w)))))) \end{aligned} \quad (47)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0v \in (ty_2Elist_2Elist \\ 2).((p\ (ap\ (c_2Ewords_2Eword_lsb\ A_27a)\ (ap\ (c_2Ebitstring_2Ev2w \\ A_27a)\ V0v))) \Leftrightarrow ((\neg(V0v = (c_2Elist_2ENIL\ 2))) \wedge (p\ (ap\ (c_2Elist_2ELAST \\ 2)\ V0v)))))) \end{aligned}$$