

thm\_2Ebitstring\_2Eword\_\_or\_\_v2w  
(TMdmhZ9Gjhd6JpttipuP9PPrTE9XjWEKFNS)

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Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elist\_2ENIL\ A.27a \in (ty\_2Elist\_2Elist\ A.27a) \quad (2)$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2)))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elist\_2ECONS\ A.27a \in (((ty\_2Elist\_2Elist\ A.27a)^{(ty\_2Elist\_2Elist\ A.27a)})^{A.27a}) \quad (3)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (4)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elist\_2ELENGTH\ A.27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A.27a)}) \quad (5)$$

Let  $c\_2Earithmetic\_2E2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

Let  $c\_2Elist\_2ETAKE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elist\_2ETAKE\ A.27a \in (((ty\_2Elist\_2Elist\ A.27a)^{(ty\_2Elist\_2Elist\ A.27a)})^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 4** We define  $c\_2Ebitstring\_2Eshiftr$  to be  $\lambda V0v \in (ty\_2Elist\_2Elist\ 2). \lambda V1m \in ty\_2Enum\_2Enum$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (10)$$

**Definition 5** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Elist\_2EDROP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2EDROP\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21\ 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 7** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b.V0x))$

Let  $c\_2Elist\_2EGENLIST : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2EGENLIST\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Enum\_2Enum)})^{(A\_27a)^{ty\_2Enum\_2Enum}}) \quad (12)$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (13)$$

**Definition 8** We define  $c\_2Elist\_2EPAD\_LEFT$  to be  $\lambda A\_27a : \iota. \lambda V0c \in A\_27a. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 9** We define  $c\_2Ebitstring\_2Ezero\_extend$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. \lambda V1v \in (ty\_2Elist\_2Elist$

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t))\ c\_2Ebool\_2E\_21$

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21\ 2)) (\lambda V2t \in$

**Definition 13** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 14** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40$

**Definition 15** We define `c_2Eprim_rec_2E_3C` to be  $\lambda V0m \in \text{ty\_2Enum\_2Enum}. \lambda V1n \in \text{ty\_2Enum\_2Enum}$

**Definition 16** We define `c_2Ebool_2ECOND` to be  $\lambda A. 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 27a. (\lambda V2t2 \in A. 27a. ($

**Definition 17** We define `c_2Ebool_2ELET` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. (\lambda V0f \in (A. 27b^{A-27a}). (\lambda V1x \in A. 27$

**Definition 18** We define `c_2Ebitstring_2Efixwidth` to be  $\lambda V0n \in \text{ty\_2Enum\_2Enum}. \lambda V1v \in (\text{ty\_2Elist\_2E}$

**Definition 19** We define `c_2Ebitstring_2Efield` to be  $\lambda V0h \in \text{ty\_2Enum\_2Enum}. \lambda V1l \in \text{ty\_2Enum\_2Enum}$

**Definition 20** We define `c_2Ebitstring_2Etestbit` to be  $\lambda V0b \in \text{ty\_2Enum\_2Enum}. \lambda V1v \in (\text{ty\_2Elist\_2E}$

Let `ty_2Efcf_2Efinite_image` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty (ty\_2Efcf\_2Efinite\_image } A0) \quad (14)$$

Let `ty_2Ebool_2Eitself` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty (ty\_2Ebool\_2Eitself } A0) \quad (15)$$

Let `c_2Ebool_2Ethe_value` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow c\_2Ebool\_2Ethe\_value \ A. 27a \in ( \quad (16)$$

$$\text{ty\_2Ebool\_2Eitself } A. 27a)$$

Let `c_2Efcf_2Edimindex` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow c\_2Efcf\_2Edimindex \ A. 27a \in (\text{ty\_2Enum\_2Enum}^{(\text{ty\_2Ebool\_2Eitself } A. 27a)}) \quad (17)$$

**Definition 21** We define `c_2Ebool_2E_3F_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap (ap c_2Ebool_2E_2F_5C$

**Definition 22** We define `c_2Efcf_2Efinite_index` to be  $\lambda A. 27a : \iota. (\text{ap (c_2Emin_2E_40 (A_27a}^{\text{ty\_2Enum\_2Enum}}$

Let `ty_2Efcf_2Ecart` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty (ty\_2Efcf\_2Ecart} \quad (18)$$

$$A0 \ A1)$$

Let `c_2Efcf_2Edest_cart` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow c\_2Efcf\_2Edest\_cart \quad (19)$$

$$A. 27a \ A. 27b \in ((A. 27a)^{(\text{ty\_2Efcf\_2Efinite\_image } A. 27b)}) (\text{ty\_2Efcf\_2Ecart } A. 27a \ A. 27b)$$

**Definition 23** We define `c_2Efcf_2Efcf_index` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0x \in (\text{ty\_2Efcf\_2Ecart } A. 27$

**Definition 24** We define  $c\_2Efc\_2EFCP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (ap$

**Definition 25** We define  $c\_2Ebitstring\_2Ev2w$  to be  $\lambda A\_27a : \iota. \lambda V0v \in (ty\_2Elist\_2Elist\ 2). (ap (c\_2Efc\_2EFCP$

**Definition 26** We define  $c\_2Earithmetic\_2EMAX$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2E$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (20)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (21)$$

**Definition 27** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2$

Let  $c\_2Elist\_2EZIP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EZIP\ A\_27a\ A\_27b \in ((ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))^{(ty\_2Epair\_2Eprod\ (ty\_2Elist\_2Elist\ A\_27a\ A\_27b))}) \quad (22)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (23)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (24)$$

**Definition 28** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in ((A\_27c^{A\_27$

Let  $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EMAP\ A\_27a\ A\_27b \in (((ty\_2Elist\_2Elist\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(A\_27b^{A\_27a})}) \quad (25)$$

**Definition 29** We define  $c\_2Ebitstring\_2Ebitwise$  to be  $\lambda V0f \in ((2^2)^2). \lambda V1v1 \in (ty\_2Elist\_2Elist\ 2). \lambda V2v$

**Definition 30** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21\ 2) (\lambda V2t \in$

**Definition 31** We define  $c\_2Ebitstring\_2Ebor$  to be  $(ap\ c\_2Ebitstring\_2Ebitwise\ c\_2Ebool\_2E\_5C\_2F)$ .

**Definition 32** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27$

**Definition 33** We define  $c\_Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_Ecombin\_2ES A\_27a (A\_27a^{A\_27a}) A$

Let  $c\_Elist\_2EHD : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Elist\_2EHD A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist A\_27a)}) \quad (26)$$

Let  $c\_Elist\_2EEL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Elist\_2EEL A\_27a \in ((A\_27a^{(ty\_2Elist\_2Elist A\_27a)})^{ty\_2Enum\_2Enum}) \quad (27)$$

**Definition 34** We define  $c\_Emarker\_2EAbbrev$  to be  $\lambda V0x \in 2.V0x$ .

Let  $c\_Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (28)$$

Let  $c\_Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (29)$$

**Definition 35** We define  $c\_Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 36** We define  $c\_Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (30)$$

**Definition 37** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 38** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2E$

Let  $c\_Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (31)$$

Let  $c\_Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (32)$$

**Definition 39** We define  $c\_Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 40** We define  $c\_2Enumeral\_2EiiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2ESUC (ap$

Let  $c\_Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (33)$$

**Definition 41** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 42** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B V0n) c\_2Enum\_2E0) = V0n$ .

**Definition 43** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B V0n) c\_2Enum\_2E0) = V0n$ .

**Definition 44** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 45** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = V1n$ .

**Definition 46** We define  $c\_2Ewords\_2Eword\_or$  to be  $\lambda A.27a : \iota.\lambda V0v \in (ty\_2EfcP\_2Ecart 2 A.27a).\lambda V1v \in (ty\_2EfcP\_2Ecart 2 A.27a).V0v \vee V1v$ .

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m)) \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & ((ap (ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap ( \\ & ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\ & (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\ & V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\ & V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)))))))))) \end{aligned} \quad (35)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B V1n) V0m)))) \quad (36)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B V1n) V0m)))) \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & \forall V2p \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\ & (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\ & (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p)))))) \end{aligned} \quad (38)$$

Assume the following.

$$(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0m)) V1n)))) \quad (39)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Enum\_2E0) V0n))) \quad (40)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m)))))) \quad (41)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) c\_2Enum\_2E0)) \Leftrightarrow (V0n = c\_2Enum\_2E0))) \quad (42)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(((ap (ap c\_2Earithmetic\_2E\_2D c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge ((ap (ap c\_2Earithmetic\_2E\_2D V0m) c\_2Enum\_2E0) = V0m))) \quad (43)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)))))))))) \quad (44)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))) \Rightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p)))))) \quad (45)$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D V0c) V0c) = c\_2Enum\_2E0)) \quad (46)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m))))))) \quad (47)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \forall V2p \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))) \quad (48)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1n)) V0m)))))) \quad (49)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1n)) V0m)))))) \quad (50)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC V0n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)) V0n))) \quad (51)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \forall V2p \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D (ap (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p) = (ap (ap c\_2Earithmetic\_2E\_2D V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)))))) \quad (52)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \forall V2p \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) (ap (ap c\_2Earithmetic\_2E\_2D V1n) V2p))) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) c\_2Enum\_2E0)))))) \quad (53)$$



Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C ( \\
& ap (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p)) \Leftrightarrow ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p))) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad c\_2Enum\_2E0) V2p)))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (\forall V1a \in ty\_2Enum\_2Enum. \\
& (\forall V2b \in ty\_2Enum\_2Enum. ((p (ap V0P (ap (ap c\_2Earithmetic\_2E\_2D \\
& V1a) V2b))) \Leftrightarrow (\forall V3d \in ty\_2Enum\_2Enum. (((V2b = (ap (ap c\_2Earithmetic\_2E\_2B \\
& V1a) V3d)) \Rightarrow (p (ap V0P c\_2Enum\_2E0))) \wedge ((V1a = (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad V2b) V3d)) \Rightarrow (p (ap V0P V3d))))))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0w \in (ty\_2Elist\_2Elist\ 2). ((ap (ap c\_2Ebitstring\_2Efixwidth \\
& (ap (c\_2Elist\_2ELENGTH\ 2) V0w)) V0w) = V0w))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1v \in (ty\_2Elist\_2Elist \\
& 2). ((ap (c\_2Elist\_2ELENGTH\ 2) (ap (ap c\_2Ebitstring\_2Efixwidth \\
& \quad V0n) V1v)) = V0n)))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0i \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2w \in (ty\_2Elist\_2Elist\ 2). ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0i) V1n)) \Rightarrow ((p (ap (ap (c\_2Elist\_2EEL\ 2) V0i) (ap (ap c\_2Ebitstring\_2Efixwidth \\
& V1n) V2w))) \Leftrightarrow (p (ap (ap (ap (c\_2Ebool\_2ECOND\ 2) (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap (c\_2Elist\_2ELENGTH\ 2) V2w)) V1n)) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2D \\
& V1n) (ap (c\_2Elist\_2ELENGTH\ 2) V2w))) V0i)) (ap (ap (c\_2Elist\_2EEL \\
& 2) (ap (ap c\_2Earithmetic\_2E\_2D V0i) (ap (ap c\_2Earithmetic\_2E\_2D \\
& V1n) (ap (c\_2Elist\_2ELENGTH\ 2) V2w)))) V2w))) (ap (ap (c\_2Elist\_2EEL \\
& 2) (ap (ap c\_2Earithmetic\_2E\_2B V0i) (ap (ap c\_2Earithmetic\_2E\_2D \\
& (ap (c\_2Elist\_2ELENGTH\ 2) V2w)) V1n))) V2w)))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty\_2Enum\_2Enum. (\forall V1v \in (ty\_2Elist\_2Elist \\
& \quad 2). ((p (ap (ap c\_2Ebitstring\_2Etestbit V0b) V1v)) \Leftrightarrow (p (ap (ap ( \\
& \quad c\_2Ebool\_2ELET ty\_2Enum\_2Enum 2) (\lambda V2n \in ty\_2Enum\_2Enum. \\
& \quad (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap c\_2Eprim\_rec\_2E\_3C V0b) V2n)) \\
& \quad (ap (ap (c\_2Elist\_2EEL 2) (ap (ap c\_2Earithmetic\_2E\_2D (ap (ap \\
& \quad c\_2Earithmetic\_2E\_2D V2n) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO)))) V0b)) V1v)))) (ap (c\_2Elist\_2ELENGTH \\
& \quad 2) V1v))))))
\end{aligned} \tag{60}$$

Assume the following.

$$True \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{62}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{63}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \tag{64}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\
& \forall V0f \in (A\_27b^{A\_27a}). (\forall V1x \in A\_27a. ((ap (ap (c\_2Ebool\_2ELET \\
& \quad A\_27a A\_27b) V0f) V1x) = (ap V0f V1x)))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
& \quad A\_27a. (p V0t)) \Leftrightarrow (p V0t)))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge \\
& \quad ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& \quad (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{71}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{72}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p V0t))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in \\
& A\_27a.(((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) \\
& V0t1) V1t2) = V1t2))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ( \\
& (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg( \\
& p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B)))))))))
\end{aligned} \tag{77}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (78)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow \text{False}) \Leftrightarrow ((p V0t) \Leftrightarrow \text{False}))) \quad (79)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (80)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2))))))) \quad (81)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ & \quad \forall V0b \in 2.(\forall V1f \in (A\_27b^{A\_27a}).(\forall V2g \in (A\_27b^{A\_27a}). \\ & \quad (\forall V3x \in A\_27a.((\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Ebool\_2ECOND } (A\_27b^{A\_27a})) \\ & \quad V0b) V1f) V2g) V3x) = (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Ebool\_2ECOND } A\_27b) V0b) (\text{ap } \\ & \quad V1f V3x)) (\text{ap } V2g V3x))))))))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ & \quad \forall V0f \in (A\_27b^{A\_27a}).(\forall V1b \in 2.(\forall V2x \in A\_27a. \\ & \quad (\forall V3y \in A\_27a.((\text{ap } V0f (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Ebool\_2ECOND } A\_27a) \\ & \quad V1b) V2x) V3y)) = (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Ebool\_2ECOND } A\_27b) V1b) (\text{ap } V0f \\ & \quad V2x)) (\text{ap } V0f V3y))))))))) \end{aligned} \quad (83)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ & \quad 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))))) \Rightarrow \\ & \quad (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \end{aligned} \quad (84)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in 2.(\forall V1P\_27 \in 2.(\forall V2Q \in 2.(\forall V3Q\_27 \in \\ & \quad 2.(((p V2Q) \Rightarrow ((p V0P) \Leftrightarrow (p V1P\_27))) \wedge ((p V1P\_27) \Rightarrow ((p V2Q) \Leftrightarrow (p V3Q\_27)))))) \Rightarrow \\ & \quad (((p V0P) \wedge (p V2Q)) \Leftrightarrow ((p V1P\_27) \wedge (p V3Q\_27)))))) \end{aligned} \quad (85)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\
& (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\
& V5y\_27)))))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a. (\forall V1t2 \in \\
& A\_27a. ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap \\
& (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V2t1)\ V3t2) = V3t2))))))
\end{aligned} \tag{87}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap\ (c\_2Ecombin\_2EI \\
A\_27a)\ V0x) = V0x)) \tag{88}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0x \in (ty\_2Efc\_2Ecart\ A\_27a\ A\_27b). (\forall V1y \in (ty\_2Efc\_2Ecart \\
& A\_27a\ A\_27b). ((V0x = V1y) \Leftrightarrow (\forall V2i \in ty\_2Enum\_2Enum. ((p\ (ap \\
& (ap\ c\_2Eprim\_rec\_2E\_3C\ V2i)\ (ap\ (c\_2Efc\_2Edimindex\ A\_27b)\ ( \\
& c\_2Ebool\_2Ethe\_value\ A\_27b)))))) \Rightarrow ((ap\ (ap\ (c\_2Efc\_2Efc\_index \\
& A\_27a\ A\_27b)\ V0x)\ V2i) = (ap\ (ap\ (c\_2Efc\_2Efc\_index\ A\_27a\ A\_27b) \\
& V1y)\ V2i))))))
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (\forall V1i \in ty\_2Enum\_2Enum. \\
& ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1i)\ (ap\ (c\_2Efc\_2Edimindex\ A\_27b) \\
& (c\_2Ebool\_2Ethe\_value\ A\_27b)))))) \Rightarrow ((ap\ (ap\ (c\_2Efc\_2Efc\_index \\
& A\_27a\ A\_27b)\ (ap\ (c\_2Efc\_2EFCP\ A\_27a\ A\_27b)\ V0g))\ V1i) = (ap\ V0g \\
& V1i))))))
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a) \\
& (c\_2Elist\_2ENIL\ A\_27a)) = c\_2Enum\_2E0) \wedge (\forall V0h \in A\_27a. ( \\
& \forall V1t \in (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (c\_2Elist\_2ELENGTH \\
& A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ c\_2Enum\_2ESUC \\
& (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1t))))))
\end{aligned} \tag{91}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& (\forall V0f \in (A.27b^{A.27a}).((ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b) \\
& V0f)\ (c.2Elist.2ENIL\ A.27a)) = (c.2Elist.2ENIL\ A.27b))) \wedge (\forall V1f \in \\
& (A.27b^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in (ty.2Elist.2Elist \\
& A.27a).((ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b)\ V1f)\ (ap\ (ap\ (c.2Elist.2ECONS \\
& A.27a)\ V2h)\ V3t)) = (ap\ (ap\ (c.2Elist.2ECONS\ A.27b)\ (ap\ V1f\ V2h)) \\
& (ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b)\ V1f)\ V3t)))))) \\
& \hspace{15em} (92)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \forall V0l \in (ty.2Elist.2Elist\ A.27a).(\forall V1f \in (A.27b^{A.27a}). \\
& ((ap\ (c.2Elist.2ELENGTH\ A.27b)\ (ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b) \\
& V1f)\ V0l)) = (ap\ (c.2Elist.2ELENGTH\ A.27a)\ V0l)))) \\
& \hspace{15em} (93)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \forall V0n \in ty.2Enum.2Enum.(\forall V1l \in (ty.2Elist.2Elist \\
& A.27a).((p\ (ap\ (ap\ c.2Eprim.rec.2E_3C\ V0n)\ (ap\ (c.2Elist.2ELENGTH \\
& A.27a)\ V1l))) \Rightarrow (\forall V2f \in (A.27b^{A.27a}).((ap\ (ap\ (c.2Elist.2EEL \\
& A.27b)\ V0n)\ (ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b)\ V2f)\ V1l)) = (ap \\
& V2f\ (ap\ (ap\ (c.2Elist.2EEL\ A.27a)\ V0n)\ V1l)))))) \\
& \hspace{15em} (94)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty.2Elist.2Elist \\
& A.27a).(((ap\ (c.2Elist.2ELENGTH\ A.27a)\ V0l) = c.2Enum.2E0) \Leftrightarrow ( \\
& V0l = (c.2Elist.2ENIL\ A.27a)))) \\
& \hspace{15em} (95)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \forall V0n \in ty.2Enum.2Enum.(\forall V1l \in A.27b.(\forall V2ls \in \\
& (ty.2Elist.2Elist\ A.27b).(((ap\ (c.2Elist.2EEL\ A.27a)\ c.2Enum.2E0) = \\
& (c.2Elist.2EHD\ A.27a)) \wedge ((ap\ (ap\ (c.2Elist.2EEL\ A.27b)\ (ap\ c.2Enum.2ESUC \\
& V0n))\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27b)\ V1l)\ V2ls)) = (ap\ (ap\ (c.2Elist.2EEL \\
& A.27b)\ V0n)\ V2ls)))))) \\
& \hspace{15em} (96)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (((ap\ (c\_2Elist\_2EZIP \\
& A\_27c\ A\_27d)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Elist\_2Elist\ A\_27c) \\
& (ty\_2Elist\_2Elist\ A\_27d))\ (c\_2Elist\_2ENIL\ A\_27c))\ (c\_2Elist\_2ENIL \\
& A\_27d))) = (c\_2Elist\_2ENIL\ (ty\_2Epair\_2Eprod\ A\_27c\ A\_27d))) \wedge \\
& (\forall V0x1 \in A\_27a. (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a). \\
& (\forall V2x2 \in A\_27b. (\forall V3l2 \in (ty\_2Elist\_2Elist\ A\_27b). \\
& ((ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Elist\_2Elist \\
& A\_27a)\ (ty\_2Elist\_2Elist\ A\_27b))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a) \\
& V0x1)\ V1l1))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V2x2)\ V3l2)))) = (ap \\
& (ap\ (c\_2Elist\_2ECONS\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ ( \\
& c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x1)\ V2x2))\ (ap\ (c\_2Elist\_2EZIP\ A\_27a \\
& A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist \\
& A\_27b))\ V1l1)\ V3l2))))))))))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V1l2 \in (ty\_2Elist\_2Elist \\
& A\_27b). (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1) = (ap\ (c\_2Elist\_2ELENGTH \\
& A\_27b)\ V1l2))) \Rightarrow (((ap\ (c\_2Elist\_2ELENGTH\ (ty\_2Epair\_2Eprod\ A\_27a \\
& A\_27b))\ (ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27b))\ V0l1)\ V1l2)))) = \\
& (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1)) \wedge ((ap\ (c\_2Elist\_2ELENGTH \\
& (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27b) \\
& (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist \\
& A\_27b))\ V0l1)\ V1l2)))) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27b)\ V1l2))))))
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V1l2 \in (ty\_2Elist\_2Elist \\
& A\_27b). (\forall V2n \in ty\_2Enum\_2Enum. (((ap\ (c\_2Elist\_2ELENGTH \\
& A\_27a)\ V0l1) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27b)\ V1l2)) \wedge (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& V2n)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1)))) \Rightarrow ((ap\ (ap\ (c\_2Elist\_2EEL \\
& (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V2n)\ (ap\ (c\_2Elist\_2EZIP\ A\_27a \\
& A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist \\
& A\_27b))\ V0l1)\ V1l2))) = (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ (ap \\
& (ap\ (c\_2Elist\_2EEL\ A\_27a)\ V2n)\ V0l1))\ (ap\ (ap\ (c\_2Elist\_2EEL\ A\_27b) \\
& V2n)\ V1l2))))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$



Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ ( \\
& ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ ( \\
& ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ ( \\
& ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap \\
& c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC \\
& (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ c\_2Earithmetic\_2EZERO)) = ( \\
& ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ ( \\
& ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow \forall A\_27c. \\
& nonempty A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}). (\forall V1x \in \\
& A\_27a. (\forall V2y \in A\_27b. ((ap (ap (c\_2Epair\_2EUNCURRY A\_27a \\
& A\_27b A\_27c) V0f) (ap (ap (c\_2Epair\_2E\_2C A\_27a A\_27b) V1x) V2y)) = \\
& (ap (ap V0f V1x) V2y))))))
\end{aligned} \tag{103}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V0n)))) \tag{104}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) c\_2Enum\_2E0)))) \tag{105}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg (\neg (p V0t))) \Leftrightarrow (p V0t))) \tag{106}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg (p V0A)) \Rightarrow False))) \tag{107}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg ((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
((p V0A) \Rightarrow False) \Rightarrow ((\neg (p V1B)) \Rightarrow False)))) \tag{108}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg ((\neg (p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
((p V0A) \Rightarrow ((\neg (p V1B)) \Rightarrow False)))) \tag{109}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (110)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\ & p V2r)) \vee (\neg(p V1q)))))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (111)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p)))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (112)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (113)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (114)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0v \in (ty\_2Elist\_2Elist \\ & 2).(\forall V1w \in (ty\_2Elist\_2Elist \ 2).((ap \ (ap \ (c\_2Ewords\_2Eword\_or \\ & A\_27a) \ (ap \ (c\_2Ebitstring\_2Ev2w \ A\_27a) \ V0v)) \ (ap \ (c\_2Ebitstring\_2Ev2w \\ & A\_27a) \ V1w)) = (ap \ (c\_2Ebitstring\_2Ev2w \ A\_27a) \ (ap \ (ap \ c\_2Ebitstring\_2Ebor \\ & V0v) \ V1w)))))) \end{aligned}$$