

thm\_2Eblast\_2EBCARRY\_\_LEM  
(TMZ1TEXbXCtvX9s6MNRbMHf6hs74i5Pgv2d)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then}$  (the  $(\lambda x. x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c\_2Emin\_2E\_40 A$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \tag{2}$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \tag{3}$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{4}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \tag{5}$$

**Definition 4** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 5** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega)^{ty\_2Enum\_2Enum} \tag{6}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega)^{\omega} \tag{7}$$

**Definition 6** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 7** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

**Definition 8** We define `c_2Enum_2ESUC` to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c_2Enum\_2EABS\_num$

Let `c_2Earithmetic_2E_2B` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 9** We define `c_2Earithmetic_2EBIT2` to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c_2Earithmetic\_2E\_2B$

**Definition 10** We define `c_2Earithmetic_2ENUMERAL` to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let `c_2Earithmetic_2EEXP` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

**Definition 11** We define `c_2Ebit_2EDIV_2EXP` to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let `c_2Earithmetic_2E_2D` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 12** We define `c_2Ebit_2EMOD_2EXP` to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 13** We define `c_2Ebit_2EBITS` to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V1m \in ty\_2Enum\_2Enum$

**Definition 14** We define `c_2Ebool_2EF` to be  $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 15** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 16** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t)))$

**Definition 17** We define `c_2Ebool_2ECOND` to be  $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (ap (c_2Ebool_2E_21 2))$

**Definition 18** We define `c_2Ebit_2ESBIT` to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap (ap (c_2Ebool_2E_21 2))$

Let `c_2Elogroot_2ELOG` :  $\iota$  be given. Assume the following.

$$c\_2Elogroot\_2ELOG \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 19** We define `c_2Ebit_2ELOG2` to be  $(ap c_2Elogroot\_2ELOG (ap c_2Earithmetic\_2ENUMERAL$

**Definition 20** We define `c_2Earithmetic_2EBIT1` to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c_2Earithmetic\_2E\_2B$

**Definition 21** We define `c_2Ebit_2EBIT` to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap (ap (c_2Ebool_2E_21 2))$

**Definition 22** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 23** We define  $c\_Eblast\_2Ebcarry$  to be  $\lambda V0x \in 2.\lambda V1y \in 2.\lambda V2c \in 2.(ap (ap c\_Ebool\_2E\_5C\_2F$

Let  $c\_Eblast\_2EBcARRY : \iota$  be given. Assume the following.

$$c\_Eblast\_2EBcARRY \in (((2^2)^{(2^{ty\_2Enum\_2Enum})})^{(2^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 24** We define  $c\_Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_2E\_3D\_3D\_3E V0t) c\_Ebool\_2E\_21 2)$

**Definition 25** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 26** We define  $c\_Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 27** We define  $c\_Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 28** We define  $c\_Enumeral\_2EiiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap c\_Enum\_2ESUC (ap$

**Definition 29** We define  $c\_Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (13)$$

Let  $c\_Enumeral\_2Eonecount : \iota$  be given. Assume the following.

$$c\_Enumeral\_2Eonecount \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (14)$$

Let  $c\_Enumeral\_2Eexactlog : \iota$  be given. Assume the following.

$$c\_Enumeral\_2Eexactlog \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \quad (15)$$

**Definition 30** We define  $c\_Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_Ebool\_2E\_21 2)$

**Definition 31** We define  $c\_Earithmetic\_2EDIV2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic\_2E\_3E$

Let  $c\_Enumeral\_2Eexp\_help : \iota$  be given. Assume the following.

$$c\_Enumeral\_2Eexp\_help \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (16)$$

Let  $c\_Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (17)$$

**Definition 32** We define  $c\_Ebool\_2ELET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27a$

Let  $c\_Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (18)$$

**Definition 33** We define  $c\_2Enumeral\_2EiDUB$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E$

**Definition 34** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 35** We define  $c\_2Enumeral\_2Einternal\_mult$  to be  $c\_2Earithmetic\_2E\_2A$ .

Assume the following.

$$\begin{aligned} ((\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\ V0m) c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\ c\_2Earithmetic\_2EZERO)))) \wedge (\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in \\ ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V1m) (ap c\_2Enum\_2ESUC \\ V2n)) = (ap (ap c\_2Earithmetic\_2E\_2A V1m) (ap (ap c\_2Earithmetic\_2EEXP \\ V1m) V2n))))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\ c\_2Enum\_2E0) = V0m)) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\ ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\ (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\ V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\ V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)))))))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\ V1n) V0m)))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\ V1n) V0m)))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ \forall V2p \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\ (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\ (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p)))))) \end{aligned} \quad (24)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((V0m = c\_2Enum\_2E0) \vee (\exists V1n \in ty\_2Enum\_2Enum.(V0m = (ap\ c\_2Enum\_2ESUC\ V1n)))))) \quad (25)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ (ap\ c\_2Enum\_2ESUC\ V0m))\ (ap\ c\_2Enum\_2ESUC\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n)))))) \quad (26)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n)) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ (ap\ c\_2Enum\_2ESUC\ V0m))\ V1n)))))) \quad (27)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ c\_2Enum\_2E0)\ V0n))) \quad (28)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V1n)\ V0m)))))) \quad (29)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ c\_2Enum\_2E0)\ V0m) = c\_2Enum\_2E0) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ c\_2Enum\_2E0) = V0m))) \quad (30)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ c\_2Enum\_2ESUC\ V0m))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) = V0m)) \quad (31)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))\ V0m) = V0m)) \quad (32)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
& (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2A (ap \\
& (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap (ap c\_2Earithmetic\_2E\_2A V0m) V2p)) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V1n) V2p))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = c\_2Enum\_2E0) \Leftrightarrow ((V0m = \\
& c\_2Enum\_2E0) \wedge (V1n = c\_2Enum\_2E0))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C ( \\
& ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p)) (ap (ap c\_2Earithmetic\_2E\_2B \\
& V1n) V2p))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
& ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\
& V0n) = (ap (ap c\_2Earithmetic\_2E\_2B V0n) V0n)))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \Rightarrow (\exists V2p \in ty\_2Enum\_2Enum. (V1n = (ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p))))))) \quad (39)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m)))))) \quad (40)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))) \quad (41)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\neg (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1n)) V0m)))) \quad (42)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\neg (V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1n)) V0m)))))) \quad (43)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)) V0n))) \quad (44)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. (\forall V2q \in ty\_2Enum\_2Enum. ((\exists V3r \in ty\_2Enum\_2Enum. (V1k = (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V2q) V0n)) V3r)) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C V3r) V0n)))) \Rightarrow (ap (ap c\_2Earithmetic\_2EDIV V1k) V0n) = V2q)))) \quad (45)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. (p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C V1k) V0n)) \Rightarrow ((ap (ap c\_2Earithmetic\_2EMOD V1k) V0n) = V1k)))))) \quad (46)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V0n)) \Rightarrow (((ap (ap c\_2Earithmetic\_2EDIV V0n) V0n) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \wedge ((ap (ap c\_2Earithmetic\_2EMOD V0n) V0n) = c\_2Enum\_2E0)))))) \quad (47)$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum. (\forall V1y \in ty\_2Enum\_2Enum. (p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap (ap c\_2Earithmetic\_2EEXP V0x) V1y))) \Leftrightarrow ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V0x)) \vee (V1y = c\_2Enum\_2E0)))))) \quad (48)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0n) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \wedge ((ap (ap c\_2Earithmetic\_2EEXP V0n) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0n)))))) \quad (49)$$

Assume the following.

$$(\forall V0b \in ty\_2Enum\_2Enum. ((p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0b)) \Rightarrow (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2m \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap (ap c\_2Earithmetic\_2EEXP V0b) V2m)) (ap (ap c\_2Earithmetic\_2EEXP V0b) V1n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C V2m) V1n)))))))))) \quad (50)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1p \in ty\_2Enum\_2Enum. ((p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V1p)) V0n)) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) (ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) (ap c\_2Enum\_2ESUC V1p)))))) \Rightarrow ((ap c\_2Ebit\_2ELOG2 V0n) = V1p)))))) \quad (51)$$



Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Enum\_2Enum. (\forall V1b \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0a) V1b)) \Rightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V0a)) (ap (ap \\
& c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\
& c\_2Earithmetic\_2EZERO))) V1b))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. ( \\
& \forall V2n \in ty\_2Enum\_2Enum. ((ap (ap (ap c\_2Ebit\_2EBITS V0h) V1l) \\
& V2n) = (ap (ap c\_2Earithmetic\_2EMOD (ap (ap c\_2Earithmetic\_2EDIV \\
& V2n) (ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V1l))) ( \\
& ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) (ap (ap c\_2Earithmetic\_2E\_2D \\
& (ap c\_2Enum\_2ESUC V0h) V1l))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. ( \\
& \forall V2n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap \\
& (ap (ap c\_2Ebit\_2EBITS V0h) V1l) V2n)) (ap (ap c\_2Earithmetic\_2EEXP \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\
& (ap (ap c\_2Earithmetic\_2E\_2D (ap c\_2Enum\_2ESUC V0h) V1l))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Ebit\_2EBIT V0b) (ap \\
& (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\
& c\_2Earithmetic\_2EZERO))) V0b)))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. ( \\
& \forall V2a \in ty\_2Enum\_2Enum. (\forall V3b \in ty\_2Enum\_2Enum. (( \\
& p (ap (ap c\_2Eprim\_rec\_2E\_3C V3b) (ap (ap c\_2Earithmetic\_2EEXP \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\
& V1l))) \Rightarrow ((ap (ap (ap c\_2Ebit\_2EBITS V0h) V1l) (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap (ap c\_2Earithmetic\_2E\_2A V2a) (ap (ap c\_2Earithmetic\_2EEXP \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\
& V1l))) V3b)) = (ap (ap (ap c\_2Ebit\_2EBITS V0h) V1l) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V2a) (ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V1l))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1a \in ty\_2Enum\_2Enum. ( \\
& \neg((ap (ap (ap c\_2Ebit\_2EBITS V0n) V0n) V1a) = (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \Leftrightarrow ((ap ( \\
& ap (ap c\_2Ebit\_2EBITS V0n) V0n) V1a) = c\_2Enum\_2E0)))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. ( \\
& \forall V2n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1l) (ap c\_2Enum\_2ESUC V0h))) \Rightarrow ((ap (ap c\_2Earithmetic\_2E\_2B ( \\
& ap (ap c\_2Ebit\_2ESBIT (ap (ap c\_2Ebit\_2EBIT (ap c\_2Enum\_2ESUC V0h)) \\
& V2n)) (ap (ap c\_2Earithmetic\_2E\_2D (ap c\_2Enum\_2ESUC V0h)) V1l))) \\
& (ap (ap (ap c\_2Ebit\_2EBITS V0h) V1l) V2n)) = (ap (ap (ap c\_2Ebit\_2EBITS \\
& (ap c\_2Enum\_2ESUC V0h)) V1l) V2n))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0i \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1n) (ap (ap c\_2Earithmetic\_2EEXP \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\
& V0i))) \Rightarrow \neg(p (ap (ap c\_2Ebit\_2EBIT V0i) V1n))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((\neg(V0n = c\_2Enum\_2E0)) \Rightarrow (p (ap ( \\
& ap c\_2Ebit\_2EBIT (ap c\_2Ebit\_2ELOG2 V0n)) V0n))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in (2^{ty\_2Enum\_2Enum}). (\forall V1y \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2c \in 2. ((p (ap (ap (ap (ap c\_2Eblast\_2EBCARRY c\_2Enum\_2E0) \\
& V0x) V1y) V2c)) \Leftrightarrow (p V2c)))) \wedge (\forall V3i \in ty\_2Enum\_2Enum. (\forall V4x \in \\
& (2^{ty\_2Enum\_2Enum}). (\forall V5y \in (2^{ty\_2Enum\_2Enum}). (\forall V6c \in \\
& 2. ((p (ap (ap (ap (ap c\_2Eblast\_2EBCARRY (ap c\_2Enum\_2ESUC V3i)) \\
& V4x) V5y) V6c)) \Leftrightarrow (p (ap (ap (ap c\_2Eblast\_2Ebcarry (ap V4x V3i)) ( \\
& ap V5y V3i)) (ap (ap (ap (ap c\_2Eblast\_2EBCARRY V3i) V4x) V5y) V6c))))))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\text{True} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{63}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{64}$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (65)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (66)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))) \quad (67)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (68)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))) \quad (69)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (70)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (71)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (72)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (73)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (74)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ A\_27a. (((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) \\ V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (75)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee ( \\ (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \end{aligned} \quad (76)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee \\ (p V0A)))))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg( \\ p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee \\ (p V1B)))))) \end{aligned} \quad (79)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow \text{False}) \Leftrightarrow ((p V0t) \Leftrightarrow \text{False}))) \quad (80)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p \\ V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2)))))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ 2. (((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))))) \Rightarrow \\ (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \end{aligned} \quad (83)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ (\forall V5y\_27 \in A\_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ V0P) V2x) V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) V1Q) V3x\_27) \\ V5y\_27)))))) \end{aligned} \quad (84)$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0)))\wedge \\
& (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n))\Rightarrow(p (ap V0P (ap c\_2Enum\_2ESUC \\
& V1n))))))\Rightarrow(\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n))))))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& (((ap c\_2Enum\_2ESUC c\_2Earithmetic\_2EZERO) = (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))\wedge((\forall V0n \in ty\_2Enum\_2Enum.((ap \\
& c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2EBIT1 V0n)) = (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)))\wedge(\forall V1n \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2EBIT2 \\
& V1n)) = (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Enum\_2ESUC V1n))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ ( \\
& ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ ( \\
& ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ ( \\
& ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap \\
& c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC \\
& (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ c\_2Earithmetic\_2EZERO)) = ( \\
& ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ ( \\
& ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((c\_2Earithmic\_2EZERO = (ap\ c\_2Earithmic\_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = c\_2Earithmic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((c\_2Earithmic\_2EZERO = (ap\ c\_2Earithmic\_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = c\_2Earithmic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = (ap\ c\_2Earithmic\_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = (ap\ c\_2Earithmic\_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = (ap\ c\_2Earithmic\_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = (ap\ c\_2Earithmic\_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
& \tag{89}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Earithmic\_2EZERO)\ (ap\ c\_2Earithmic\_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Earithmic\_2EZERO) \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& V0n)\ c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& (ap\ c\_2Earithmic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& (ap\ c\_2Earithmic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow \\
& (\neg(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V1m))))))))) \\
& \tag{90}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ c\_2Earithmic\_2EZERO)\ V0n)) \Leftrightarrow \\
& True) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT1 \\
& V0n))\ c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))\ c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& V0n)\ V1m))))))))) \\
& \tag{91}
\end{aligned}$$



Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))) \wedge ((ap\ c\_2Enumeral\_2EiDUB\ c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO)))) \\
& \hspace{15em} (92)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p\ (ap\ c\_2Earithmetic\_2EVEN\ c\_2Earithmetic\_2EZERO)) \wedge \\
& \quad ((p\ (ap\ c\_2Earithmetic\_2EVEN\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n))) \wedge \\
& \quad ((\neg(p\ (ap\ c\_2Earithmetic\_2EVEN\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)))) \wedge \\
& \quad \quad ((\neg(p\ (ap\ c\_2Earithmetic\_2EODD\ c\_2Earithmetic\_2EZERO)))) \wedge ((\neg(p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n)))) \wedge \\
& \quad \quad (p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))))))))) \\
& \hspace{15em} (93)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Enumeral\_2Eonecount\ c\_2Earithmetic\_2EZERO\ V0x) = V0x)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V2x \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Enumeral\_2Eonecount\ (ap\ c\_2Earithmetic\_2EBIT1\ V1n))\ V2x) = (ap\ (ap\ c\_2Enumeral\_2Eonecount\ V1n)\ (ap\ c\_2Enum\_2ESUC\ V2x)))))) \wedge ((\forall V3n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V4x \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Enumeral\_2Eonecount\ (ap\ c\_2Earithmetic\_2EBIT2\ V3n))\ V4x) = c\_2Earithmetic\_2EZERO)))))) \\
& \hspace{15em} (94)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (((ap\ c\_2Enumeral\_2Exactlog\ c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO) \wedge \\
& \quad ((\forall V0n \in ty\_2Enum\_2Enum. ((ap\ c\_2Enumeral\_2Exactlog\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) = c\_2Earithmetic\_2EZERO)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum. \\
& \quad ((ap\ c\_2Enumeral\_2Exactlog\ (ap\ c\_2Earithmetic\_2EBIT2\ V1n)) = (ap\ (ap\ (c\_2Ebool\_2ELET\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ ty\_2Enum\_2Enum)\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ ty\_2Enum\_2Enum)\ V2x)\ c\_2Earithmetic\_2EZERO))\ c\_2Earithmetic\_2EZERO)\ (ap\ c\_2Earithmetic\_2EBIT1\ V2x))))\ (ap\ c\_2Enumeral\_2Eonecount\ V1n)\ c\_2Earithmetic\_2EZERO)))))) \\
& \hspace{15em} (95)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1x \in ty\_2Enum\_2Enum. ( \\
& \forall V2y \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmic\_2E\_2A c\_2Earithmic\_2EZERO) \\
& V0n) = c\_2Earithmic\_2EZERO) \wedge (((ap (ap c\_2Earithmic\_2E\_2A \\
& V0n) c\_2Earithmic\_2EZERO) = c\_2Earithmic\_2EZERO) \wedge ((ap \\
& (ap c\_2Earithmic\_2E\_2A (ap c\_2Earithmic\_2EBIT1 V1x)) (ap \\
& c\_2Earithmic\_2EBIT1 V2y)) = (ap (ap c\_2Enumeral\_2Einternal\_mult \\
& (ap c\_2Earithmic\_2EBIT1 V1x)) (ap c\_2Earithmic\_2EBIT1 V2y)))) \wedge \\
& (((ap (ap c\_2Earithmic\_2E\_2A (ap c\_2Earithmic\_2EBIT1 V1x)) \\
& (ap c\_2Earithmic\_2EBIT2 V2y)) = (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum \\
& ty\_2Enum\_2Enum) (\lambda V3n \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2ECOND \\
& ty\_2Enum\_2Enum) (ap c\_2Earithmic\_2EODD V3n)) (ap (ap c\_2Enumeral\_2Eexp\_help \\
& (ap c\_2Earithmic\_2EDIV2 V3n)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT1 \\
& V1x)))) (ap (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmic\_2EBIT1 \\
& V1x)) (ap c\_2Earithmic\_2EBIT2 V2y)))))) (ap c\_2Enumeral\_2Eexactlog \\
& (ap c\_2Earithmic\_2EBIT2 V2y)))) \wedge (((ap (ap c\_2Earithmic\_2E\_2A \\
& (ap c\_2Earithmic\_2EBIT2 V1x)) (ap c\_2Earithmic\_2EBIT1 V2y)) = \\
& (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) (\lambda V4m \in \\
& ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) \\
& (ap c\_2Earithmic\_2EODD V4m)) (ap (ap c\_2Enumeral\_2Eexp\_help \\
& (ap c\_2Earithmic\_2EDIV2 V4m)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT1 \\
& V2y)))) (ap (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmic\_2EBIT2 \\
& V1x)) (ap c\_2Earithmic\_2EBIT1 V2y)))))) (ap c\_2Enumeral\_2Eexactlog \\
& (ap c\_2Earithmic\_2EBIT2 V1x)))) \wedge ((ap (ap c\_2Earithmic\_2E\_2A \\
& (ap c\_2Earithmic\_2EBIT2 V1x)) (ap c\_2Earithmic\_2EBIT2 V2y)) = \\
& (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) (\lambda V5m \in \\
& ty\_2Enum\_2Enum. (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) \\
& (\lambda V6n \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) \\
& (ap c\_2Earithmic\_2EODD V5m)) (ap (ap c\_2Enumeral\_2Eexp\_help \\
& (ap c\_2Earithmic\_2EDIV2 V5m)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT2 \\
& V2y)))) (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) (ap c\_2Earithmic\_2EODD \\
& V6n)) (ap (ap c\_2Enumeral\_2Eexp\_help (ap c\_2Earithmic\_2EDIV2 \\
& V6n)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT2 V1x)))) \\
& (ap (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmic\_2EBIT2 \\
& V1x)) (ap c\_2Earithmic\_2EBIT2 V2y)))))) (ap c\_2Enumeral\_2Eexactlog \\
& (ap c\_2Earithmic\_2EBIT2 V2y)))))) (ap c\_2Enumeral\_2Eexactlog \\
& (ap c\_2Earithmic\_2EBIT2 V1x))))))))) (96)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Enumeral\_2Einternal\_mult c\_2Earithmetic\_2EZERO) \\
V0n) = c\_2Earithmetic\_2EZERO) \wedge (((ap (ap c\_2Enumeral\_2Einternal\_mult \\
& V0n) c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO) \wedge ((ap \\
& (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmetic\_2EBIT1 \\
V0n)) V1m) = (ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Enumeral\_2EiDUB (ap (ap c\_2Enumeral\_2Einternal\_mult \\
V0n) V1m))) V1m))) \wedge ((ap (ap c\_2Enumeral\_2Einternal\_mult (ap \\
& c\_2Earithmetic\_2EBIT2 V0n)) V1m) = (ap c\_2Enumeral\_2EiDUB (ap \\
c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Enumeral\_2Einternal\_mult \\
& V0n) V1m)) V1m)))))))))
\end{aligned} \tag{97}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) c\_2Enum\_2E0)))) \tag{98}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Enum\_2ESUC V0n)))) \tag{99}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{100}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{101}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \tag{102}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \tag{103}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{104}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p)))))))))
\end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{106}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{107}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ( \\
& (\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{108}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{109}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{110}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{111}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{112}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{113}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{114}$$

**Theorem 1**

$(\forall V0i \in ty\_2Enum\_2Enum.(\forall V1x \in ty\_2Enum\_2Enum.($   
 $\forall V2y \in ty\_2Enum\_2Enum.(\forall V3c \in 2.((p (ap (ap c\_2Eprim\_rec\_2E\_3C$   
 $c\_2Enum\_2E0) V0i)) \Rightarrow ((p (ap (ap (ap (ap c\_2Eblast\_2EBCARRY V0i)$   
 $(\lambda V4i \in ty\_2Enum\_2Enum.(ap (ap c\_2Ebit\_2EBIT V4i) V1x))) (\lambda V5i \in$   
 $ty\_2Enum\_2Enum.(ap (ap c\_2Ebit\_2EBIT V5i) V2y))) V3c) \Leftrightarrow (p (ap$   
 $(ap c\_2Ebit\_2EBIT V0i) (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2B$   
 $(ap (ap (ap c\_2Ebit\_2EBITS (ap (ap c\_2Earithmetic\_2E\_2D V0i) (ap$   
 $c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))$   
 $c\_2Enum\_2E0) V1x)) (ap (ap (ap c\_2Ebit\_2EBITS (ap (ap c\_2Earithmetic\_2E\_2D$   
 $V0i) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1$   
 $c\_2Earithmetic\_2EZERO)))) c\_2Enum\_2E0) V2y))) (ap (ap (ap (c\_2Ebool\_2ECOND$   
 $ty\_2Enum\_2Enum) V3c) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1$   
 $c\_2Earithmetic\_2EZERO))) c\_2Enum\_2E0))))))))))$