

thm_2Ebool_2EAND__CONG (TMJhBLFmot- PUETzDMAzutCVf6yRx3ik9ETo)

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Definition 1 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A \cdot 27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A \cdot 27a}))))$

Definition 5 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \Rightarrow V0t1) \Rightarrow (p \Rightarrow V1t2)) \Rightarrow (((p \Rightarrow V1t2) \Rightarrow (p \Rightarrow V0t1)) \Rightarrow ((p \Rightarrow V0t1) \Leftrightarrow (p \Rightarrow V1t2)))))) \quad (1)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p \Rightarrow V0t1) \Rightarrow ((p \Rightarrow V1t2) \Rightarrow (p \Rightarrow V2t3))) \Leftrightarrow (((p \Rightarrow V0t1) \wedge (p \Rightarrow V1t2)) \Rightarrow (p \Rightarrow V2t3)))))) \quad (2)$$

Theorem 1

$$\begin{aligned} & (\forall V0P \in 2. (\forall V1P_27 \in 2. (\forall V2Q \in 2. (\forall V3Q_27 \in \\ & 2. (((p \Rightarrow V2Q) \Rightarrow ((p \Rightarrow V0P) \Leftrightarrow (p \Rightarrow V1P_27))) \wedge ((p \Rightarrow V1P_27) \Rightarrow ((p \Rightarrow V2Q) \Leftrightarrow (p \Rightarrow V3Q_27)))) \Rightarrow \\ & (((p \Rightarrow V0P) \wedge (p \Rightarrow V2Q)) \Leftrightarrow ((p \Rightarrow V1P_27) \wedge (p \Rightarrow V3Q_27)))))) \end{aligned}$$