

thm_2Ebool_2ECOND_EXPAND_IMP (TMb- sinbyeupkgAzuCGiE58KW7mARwFcnTY)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_{27a}}))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (P \Rightarrow Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\text{c_2Ebool_2EF } V0t))$

Definition 7 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))$

Definition 8 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))$

Definition 9 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\lambda x. x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$.

Definition 10 We define `c_2Ebool_2ECOND` to be $\lambda A_{27a} : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_{27a}. (\lambda V2t2 \in A_{27a}. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_{27a}}))$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg \text{True}) \Leftrightarrow \text{False}) \wedge ((\neg \text{False}) \Leftrightarrow \text{True}))) \quad (1)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (2)$$

Assume the following.

$$(\forall V0b \in 2. (\forall V1t1 \in 2. (\forall V2t2 \in 2. ((p (\text{ap } (\text{ap } (\text{c_2Ebool_2ECOND } 2) V0b) V1t1) V2t2)) \Leftrightarrow (((\neg(p V0b)) \vee (p V1t1)) \wedge ((p V0b) \vee (p V2t2)))))) \quad (3)$$

Theorem 1

$$\begin{aligned} & (\forall V0b \in 2. (\forall V1t1 \in 2. (\forall V2t2 \in 2. ((p \text{ (ap (ap} \\ & (\text{ap (c.2Ebool_2ECOND 2) V0b) V1t1) V2t2)) \Leftrightarrow (((p \text{ V0b}) \Rightarrow (p \text{ V1t1})) \wedge \\ & ((\neg(p \text{ V0b})) \Rightarrow (p \text{ V2t2}))))))) \end{aligned}$$