thm_2Ebool_2EEXISTS__UNIQUE__REFL (TMZdVRpU7mjuZc3o22JHpunoN8oqwge4vKt)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 6 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 7 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Definition 8 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ c_2Ebool_2E_2F_5C

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((p\ (ap\ (\text{c\_2Ebool\_2E\_3F\_21}\ A\_27a)\ (\lambda V1x \in A\_27a.(ap\ V0P\ V1x)))) \Leftrightarrow ((\exists V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \wedge (\forall V3x \in A\_27a.(\forall V4y \in A\_27a.(((p\ (ap\ V0P\ V3x)) \wedge (p\ (ap\ V0P\ V4y))) \Rightarrow (V3x = V4y))))))) \tag{1}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in A\_27a.(\exists V1x \in A\_27a.(V1x = V0a))) \tag{2}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in A\_27a.(p\ (ap\ (\text{c\_2Ebool\_2E\_3F\_21}\ A\_27a)\ (\lambda V1x \in A\_27a.(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ A\_27a)\ V1x)\ V0a)))))$$