

thm_2Ebool_2EEXISTS__UNIQUE__THM
(TME7TkNw5J9URBiXaP3frggDmx9VKwjQci)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 6 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 8 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C A_27a P))))$

Theorem 1

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((p (ap (c_2Ebool_2E_3F_21 A_27a) (\lambda V1x \in A_27a.(ap V0P V1x)))) \Leftrightarrow ((\exists V2x \in A_27a.(p (ap V0P V2x))) \wedge (\forall V3x \in A_27a.(\forall V4y \in A_27a.((p (ap V0P V3x)) \wedge (p (ap V0P V4y))) \Rightarrow (V3x = V4y)))))))$$