

# thm\_2Ebool\_2EEXISTS\_itself (TMaN- sjNpTEnd1o2NUhiD1wEEwDkcGJfA1LK)

October 26, 2020

**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x)) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2E_3F` to be  $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } V0P \text{ (ap } (c_2Emin_2E_40 \ A_{27a} \ V0P))$

**Definition 4** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota$ .

Let `ty_2Ebool_2Eitself` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty\_2Ebool\_2Eitself } A0) \quad (1)$$

Let `c_2Ebool_2Ethe\_value` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow c_2Ebool_2Ethe\_value \ A_{27a} \in (\text{ty\_2Ebool\_2Eitself } A_{27a}) \quad (2)$$

**Definition 5** We define `c_2Ebool_2ET` to be  $(\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2. V0x)) \ (\lambda V1x \in 2. V1x))$

**Definition 6** We define `c_2Ebool_2E_21` to be  $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^{A_{27a}})) \ V0P))$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (3)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0i \in (\text{ty\_2Ebool\_2Eitself } A_{27a}). (V0i = (c_2Ebool_2Ethe\_value \ A_{27a}))) \quad (4)$$

**Theorem 1**

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in (2^{(\text{ty\_2Ebool\_2Eitself } A_{27a})}). ((\exists V1x \in (\text{ty\_2Ebool\_2Eitself } A_{27a}). (p \ (\text{ap } V0P \ V1x))) \Leftrightarrow (p \ (\text{ap } V0P \ (c_2Ebool_2Ethe\_value \ A_{27a}))))))$$