

thm_2Ebool_2EITSELF__UNIQUE (TMWYWm- PeYJ4JaVSuRg1GqVQQDnfqFSoRjts)

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Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (1)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (2)$$

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1P \in 2.V1P))\ (\lambda V2P \in 2.V2P)))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t)))$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (3)$$

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ (\lambda V1P \in 2.V1P))))$

Definition 8 We define $c_2Ebool_2ETYPE_DEFINITION$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0P \in (2^{A_27a}).(\lambda V1P \in 2.V1P))$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\exists V0rep \in (A_27a^{(ty_2Ebool_2Eitself\ A_27a)}). \\ & (p\ (ap\ (ap\ (c_2Ebool_2ETYPE_DEFINITION\ A_27a\ (ty_2Ebool_2Eitself\ A_27a))\ (ap\ (c_2Emin_2E_3D\ A_27a)\ (c_2Ebool_2EARB\ A_27a)))\ V0rep))) \end{aligned} \quad (4)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0i \in (ty_2Ebool_2Eitself\ A_27a).(V0i = (c_2Ebool_2Ethe_value\ A_27a)))$$