

thm_2Ebool_2EMONO_IMP (TMEgRjE5JqerXQ9KFsesUsREGQ5kSzdJeaB)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p \Rightarrow V0t1) \Rightarrow \\ & ((p \Rightarrow V1t2) \Rightarrow (p \Rightarrow V2t3)))) \Leftrightarrow (((p \Rightarrow V0t1) \wedge (p \Rightarrow V1t2)) \Rightarrow (p \Rightarrow V2t3)))) \end{aligned} \quad (1)$$

Theorem 1

$$\begin{aligned} & (\forall V0y \in 2. (\forall V1x \in 2. (\forall V2z \in 2. (\forall V3w \in \\ & 2. (((((p \Rightarrow V0y) \Rightarrow (p \Rightarrow V1x)) \wedge ((p \Rightarrow V2z) \Rightarrow (p \Rightarrow V3w))) \Rightarrow (((p \Rightarrow V1x) \Rightarrow (p \Rightarrow V2z)) \Rightarrow \\ & ((p \Rightarrow V0y) \Rightarrow (p \Rightarrow V3w)))))))) \end{aligned}$$