

thm\_2Ebool\_2ENOT\_EXISTS\_THM  
(TMVJ3iuZ6AqwwzVfLDXMf21fAcx19TbHuf5)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x) \text{ of type } \iota \Rightarrow \iota.$

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota.$

**Definition 3** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40 } A \ 27a))$

**Definition 4** We define `c_2Ebool_2ET` to be  $(\text{ap (ap (c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 5** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap (ap (c_2Emin_2E_3D } (2^{A-27a})$

**Definition 6** We define `c_2Ebool_2EF` to be  $(\text{ap (c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t)).$

**Definition 7** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota.$

**Definition 8** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2. (\text{ap (ap c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2EF } V0t))$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (1)$$

**Theorem 1**

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow (\forall V0P \in (2^{A-27a}). ((\neg (\exists V1x \in A. 27a. (p \text{ (ap } V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A. 27a. (\neg (p \text{ (ap } V0P \ V2x))))))$$