

thm_2Ebool_2EONTO__THM (TMWntyBuoD- cMdFfpmmodd24Rf47CSpHPLFj)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x))$
of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. \lambda 27a : \iota. (\lambda V0P \in (2^{A \cdot 27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40 } A \ 27a))$

Definition 4 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) \ (\lambda V0x \in 2. V0x)) \ (\lambda V1x \in 2. V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda 27a : \iota. (\lambda V0P \in (2^{A \cdot 27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A \cdot 27a})$

Definition 6 We define `c_2Ebool_2EONTO` to be $\lambda A. \lambda 27a : \iota. \lambda A. \lambda 27b : \iota. (\lambda V0f \in (A. 27b^{A \cdot 27a}). (\text{ap } (\text{c_2Ebool_2E_3F } A \ 27a \ 27b))$

Theorem 1

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow (\forall V0f \in (A. 27b^{A \cdot 27a}). ((p \ (\text{ap } (\text{c_2Ebool_2EONTO } A. 27a \ A. 27b) \ V0f)) \Leftrightarrow (\forall V1y \in A. 27b. (\exists V2x \in A. 27a. (V1y = (\text{ap } V0f \ V2x)))))))$$