

# thm\_2Ebool\_2EPULL\_FORALL (TMHaXNxtMiu2ysgfXATwEpVco1JrSZ3K8s1)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\ 2.(((\forall V2x \in A\_27a.(p (ap V0P V2x))) \wedge (p V1Q))) \Leftrightarrow (\forall V3x \in \\ A\_27a.((p (ap V0P V3x)) \wedge (p V1Q)))))) \end{aligned} \quad (1)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A\_27a}).(((p V0P) \wedge (\forall V2x \in A\_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in \\ A\_27a.((p V0P) \wedge (p (ap V1Q V3x)))))) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A\_27a}).(((\forall V2x \in A\_27a.((p V0P) \Rightarrow (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ V0P) \Rightarrow (\forall V3x \in A\_27a.(p (ap V1Q V3x)))))) \end{aligned} \quad (3)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\ 2.(((p V1Q) \Rightarrow (\forall V2x \in A\_27a.(p (ap V0P V2x)))) \Leftrightarrow (\forall V3x \in \\ A\_27a.((p V1Q) \Rightarrow (p (ap V0P V3x)))))) \wedge (((\forall V4x \in A\_27a.(p ( \\ ap V0P V4x))) \wedge (p V1Q)) \Leftrightarrow (\forall V5x \in A\_27a.((p (ap V0P V5x)) \wedge (p \\ V1Q)))) \wedge (((p V1Q) \wedge (\forall V6x \in A\_27a.(p (ap V0P V6x)))) \Leftrightarrow (\forall V7x \in \\ A\_27a.((p V1Q) \wedge (p (ap V0P V7x)))))) \end{aligned}$$