

thm_2Ebool_2ERES_EXISTS_CONG (TM- PqTJKEkGKEMQ7rugGy2aT8pbLW5k65jPB)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2EIN` to be $\lambda A. \lambda a : \iota. (\lambda V0x \in A. \lambda a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f \ V0x)))$

Definition 3 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \ P \Rightarrow \ p \ Q)$ of type ι .

Definition 4 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \ (\text{ap } P \ x)) \ \text{then } (\text{the } (\lambda x. x \in A \wedge p \ (\text{ap } P \ x)))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2E_3F` to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \ (\text{ap } (\text{c_2Emin_2E_40 } A \ a))))$

Definition 9 We define `c_2Ebool_2ERES_EXISTS` to be $\lambda A. \lambda a : \iota. (\lambda V0p \in (2^{A-27a}). (\lambda V1m \in (2^{A-27a}). (\text{ap } (\text{c_2Emin_2E_40 } A \ a) \ V0p \ V1m))))$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \tag{1}$$

Theorem 1

$$\begin{aligned} & \forall A. \lambda a. \text{nonempty } A. \lambda a. \Rightarrow (\forall V0P \in (2^{A-27a}). (\forall V1Q \in (2^{A-27a}). (\forall V2f \in (2^{A-27a}). (\forall V3g \in (2^{A-27a}). (\\ & (V0P = V1Q) \Rightarrow ((\forall V4x \in A. \lambda a. ((p \ (\text{ap } (\text{ap } (\text{c_2Ebool_2EIN } A \ a) \ V4x) \ V1Q)) \Rightarrow ((p \ (\text{ap } V2f \ V4x)) \Leftrightarrow (p \ (\text{ap } V3g \ V4x)))))) \Rightarrow ((p \ (\text{ap } (\text{ap } (\text{c_2Ebool_2ERES_EXISTS } A \ a) \ V0P) \ V2f)) \Leftrightarrow (p \ (\text{ap } (\text{ap } (\text{c_2Ebool_2ERES_EXISTS } A \ a) \ V1Q) \ V3g)))))))))) \end{aligned}$$