

thm_2Ebool_2ERES_EXISTS_THM
(TMSA8AEFZdoXDXMzkXxYoBHqVrpKd9HFWGs)

October 26, 2020

Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Definition 4 We define `c_2Ebool_2EIN` to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (\text{ap } V1f V0x)))$

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p (\text{ap } P x)))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } A_27a))))$

Definition 9 We define `c_2Ebool_2ERES_EXISTS` to be $\lambda A_27a : \iota. (\lambda V0p \in (2^{A_27a}). (\lambda V1m \in (2^{A_27a}). (\text{ap } (\text{c_2Ebool_2E_3F } A_27a) V0p) V1m)))$

Theorem 1

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1f \in (2^{A_27a}). ((p (\text{ap } (\text{ap } (\text{c_2Ebool_2ERES_EXISTS } A_27a) V0P) V1f))) \Leftrightarrow (\exists V2x \in A_27a. ((p (\text{ap } (\text{ap } (\text{c_2Ebool_2EIN } A_27a) V2x) V0P)) \wedge (p (\text{ap } V1f V2x))))))))$$