

# thm\_2Ebool\_2ERES\_EXISTS\_UNIQUE\_THM (TMXb5P5nTdCVYoGNswR52mrBQh7r3ZN7eeG)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2ET` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A\_27a}))$

**Definition 4** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota$ .

**Definition 5** We define `c_2Ebool_2EIN` to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (\text{ap } V1f V0x)))$

**Definition 6** We define `c_2Ebool_2ERES_FORALL` to be  $\lambda A\_27a : \iota. (\lambda V0p \in (2^{A\_27a}). (\lambda V1m \in (2^{A\_27a}). (\text{ap } V1m$

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V2t \in 2. V2t)))$

**Definition 8** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (the (\lambda x. x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define `c_2Ebool_2E_3F` to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (\text{ap } V0P (\text{ap } (\text{c\_2Emin\_2E\_40 } A\_27a) P)))$

**Definition 10** We define `c_2Ebool_2ERES_EXISTS` to be  $\lambda A\_27a : \iota. (\lambda V0p \in (2^{A\_27a}). (\lambda V1m \in (2^{A\_27a}). (\text{ap } V1m$

**Definition 11** We define `c_2Ebool_2ERES_EXISTS_UNIQUE` to be  $\lambda A\_27a : \iota. (\lambda V0p \in (2^{A\_27a}). (\lambda V1m \in (2^{A\_27a}). (\text{ap } V1m$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1f \in \\ & (2^{A\_27a}). ((p (\text{ap } (\text{ap } (\text{c\_2Ebool\_2ERES\_EXISTS\_UNIQUE } A\_27a) \\ & V0P) V1f)) \Leftrightarrow ((p (\text{ap } (\text{ap } (\text{c\_2Ebool\_2ERES\_EXISTS } A\_27a) V0P) (\lambda V2x \in \\ & A\_27a. (\text{ap } V1f V2x)))) \wedge (p (\text{ap } (\text{ap } (\text{c\_2Ebool\_2ERES\_FORALL } A\_27a) \\ & V0P) (\lambda V3x \in A\_27a. (\text{ap } (\text{ap } (\text{c\_2Ebool\_2ERES\_FORALL } A\_27a) V0P) \\ & (\lambda V4y \in A\_27a. (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D\_3D\_3E } (\text{ap } (\text{ap } (\text{c\_2Ebool\_2E\_2F\_5C} \\ & (\text{ap } V1f V3x)) (\text{ap } V1f V4y)))) (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } A\_27a) V3x) V4y)))))))))) \end{aligned}$$