thm_2Ebool_2ERES__FORALL__CONG (TMXzVnq5CRePaw48psCdaiXvzHsjb4KACei)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A_27}$

Definition 6 *We define* c_2Ebool_2ERES__FORALL *to be* $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).(a$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (1)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).(\forall V2f \in (2^{A\_27a}).(\forall V3g \in (2^{A\_27a}).((V0P = V1Q) \Rightarrow ((\forall V4x \in A\_27a.((p\ (ap\ (ap\ (\text{c\_2Ebool\_2EIN}\ A\_27a)\ V4x)\ V1Q)) \Rightarrow ((p\ (ap\ V2f\ V4x)) \Leftrightarrow (p\ (ap\ V3g\ V4x))))) \Rightarrow ((p\ (ap\ (ap\ (\text{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ V0P)\ V2f)) \Leftrightarrow (p\ (ap\ (ap\ (\text{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ V1Q)\ V3g)))))))))$$