

# thm\_2Ebool\_2ERES\_FORALL\_TRUE (TM-MxyHbdFwhY8RcguKZJvuDo2QS5QgX1msa)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

**Definition 4** We define `c_2Ebool_2EF` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

**Definition 8** We define `c_2Ebool_2EIN` to be  $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

**Definition 9** We define `c_2Ebool_2ERES_FORALL` to be  $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).(ap$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (1)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1f \in \\ & (2^{A_27a}).((p (ap (ap (c_2Ebool_2ERES_FORALL A_27a) V0P) V1f)) \Leftrightarrow \\ & (\forall V2x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V0P)) \Rightarrow \\ & (p (ap V1f V2x)))))) \end{aligned} \quad (3)$$

**Theorem 1**

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V_0 P \in (2^{A_{27a}}).((p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL\ A_{27a})\ V_0P)\ (\lambda V_1 x \in A_{27a}.c\_2Ebool\_2ET))) \Leftrightarrow True)))$$