

thm\_2Ebool\_2Eliteral\_case\_id (TMd-HUeC9BJSAYEgGnfAVbfWvk6hgH7nfdVJ)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V0t \in 2.V0t)) (\lambda V1t \in 2.V1t)))$

**Definition 5** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p \text{ of type } \iota \Rightarrow \iota))$

**Definition 9** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (c\_2Ebool\_2E\_21 2) (\lambda V3t3 \in 2.V3t3))))))$

**Definition 10** We define  $c\_2Ebool\_2Eliteral\_case$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27a.((ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) V0t1) V1t2) = V1t2))))$

Assume the following.

$$\begin{aligned} & \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. (((ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) \\ & V0t1) V1t2) = V1t2))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a. \text{nonempty } A\_27a \Rightarrow \forall A\_27b. \text{nonempty } A\_27b \Rightarrow \\ & \forall V0f \in (A\_27b^{A\_27a}). (\forall V1x \in A\_27a. ((ap (ap (c\_2Ebool\_2Eliteral\_case \\ & A\_27a A\_27b) V0f) V1x) = (ap V0f V1x)))) \end{aligned} \tag{2}$$

**Theorem 1**

$$\begin{aligned} \forall A_{\_27a}.nonempty\ A_{\_27a} \Rightarrow & \forall A_{\_27b}.nonempty\ A_{\_27b} \Rightarrow ( \\ & \forall V0a \in A_{\_27a}.(\forall V1t \in A_{\_27b}.(\forall V2u \in A_{\_27b}.(( \\ & ap\ (ap\ (c_{\_2Ebool\_2Eliteral\_case}\ A_{\_27a}\ A_{\_27b})\ (\lambda V3x \in A_{\_27a}. \\ & (ap\ (ap\ (ap\ (c_{\_2Ebool\_2ECOND}\ A_{\_27b})\ (ap\ (ap\ (c_{\_2Emin\_2E\_3D}\ A_{\_27a}) \\ & V3x)\ V0a))\ V1t)\ V2u)))\ V0a) = V1t)))) \end{aligned}$$