thm_2Ecanonical_2Einterp__m__ok (TMRfdRAiFqPoYXR2nxixZxw44iJWbLqqfbt)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Equote\_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Equote\_2Eindex \tag{1}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{2}$$

Let $ty\_2Equote\_2Evarmap : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Equote\_2Evarmap\ A0) \tag{3}$$

Let $ty\_2Esemi\_\_ring\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A0) \tag{4}$$

Let $c\_2Ecanonical\_2Einterp\_\_vl : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a \in (((A\_27a^{(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex)})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{5}$$

Let $c\_2Ecanonical\_2Eivl\_\_aux : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Eivl\_\_aux\ A\_27a \in ((((A\_27a^{(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex)})^{ty\_2Equote\_2Eindex})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2}$$ (6)

Let $c\_2Ecanonical\_2Einterp\_\_m : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Einterp\_\_m\ A\_27a \in ((((A\_27a^{(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex)})^{A\_27a})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring}$$ (7)

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{8}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{9}$$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{10}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{11}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{12}$$

Let $c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR0\ A\_27a \in (A\_27a^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{13}$$

Definition 6 *We define* c_2Esemi__ring_2Eis__semi__ring *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Esemi\_\_ring\_2Esemi$

Assume the following.

$$True \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a)\ V0sr)\ V1vm)\ (c\_2Elist\_2ENIL\ ty\_2Equote\_2Eindex)) = (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ V0sr)))) \wedge (\forall V2sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V3vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V4x \in ty\_2Equote\_2Eindex.(\forall V5t \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a)\ V2sr)\ V3vm)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Equote\_2Eindex)\ V4x)\ V5t)) = (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eivl\_\_aux\ A\_27a)\ V2sr)\ V3vm)\ V4x)\ V5t)))))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V2c \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_m\ A\_27a)\ V0sr)\ V1vm)\ V2c)\ (c\_2Elist\_2ENIL\ ty\_2Equote\_2Eindex)) = V2c)))) \wedge (\forall V3sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).(\forall V4vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V5c \in A\_27a.(\forall V6x \in ty\_2Equote\_2Eindex.(\forall V7t \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_m\ A\_27a)\ V3sr)\ V4vm)\ V5c)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Equote\_2Eindex)\ V6x)\ V7t)) = (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ V3sr)\ V5c)\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Eivl\_\_aux\ A\_27a)\ V3sr)\ V4vm)\ V6x)\ V7t)))))))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).(((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).((p\ (ap\ (\mathsf{c\_2Esemi\_\_ring\_2Eis\_\_semi\_\_ring}\ A\_27a)\ V0r)) \Rightarrow (\forall V1n \in A\_27a.((ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ V0r)\ V1n)\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SR1\ A\_27a)\ V0r)) = V1n)))) \tag{19}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sr \in (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a).((p\ (ap\ (\mathsf{c\_2Esemi\_\_ring\_2Eis\_\_semi\_\_ring}\ A\_27a)\ V0sr)) \Rightarrow (\forall V1vm \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V2x \in A\_27a. (\forall V3l \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).((ap\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_m\ A\_27a)\ V0sr)\ V1vm)\ V2x)\ V3l) = (ap\ (ap\ (ap\ (c\_2Esemi\_\_ring\_2Esemi\_\_ring\_\_SRM\ A\_27a)\ V0sr)\ V2x)\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a)\ V0sr)\ V1vm)\ V3l))))))))$$