thm_2Ecanonical_2Eivl__aux__ok (TMY-CxW5miXaQKaURu1UhfkWpSaLewxpbJsD)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj__o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty_2Equote_2Evarmap : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Equote_2Evarmap\ A0) \tag{1}$$

Let $ty_2Equote_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty_2Equote_2Eindex \tag{2}$$

Let $c_2Equote_2Evarmap__find : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Equote_2Evarmap__find\ A_27a \in ((A_27a^{(ty_2Equote_2Evarmap\ A_27a)})^{ty_2Equote_2Eindex}) \tag{3}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{4}$$

Let $ty_2Esemi__ring_2Esemi__ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Esemi__ring_2Esemi__ring\ A0) \tag{5}$$

Let $c_2Ecanonical_2Eivl__aux : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ecanonical_2Eivl__aux\ A_27a \in ((((A_27a^{(ty_2Elist_2Elist\ ty_2Equote_2Eindex)})^{ty_2Equote_2Eindex})^{(ty_2Equote_2Evarmap\ A_27a)})^{(ty_2Esemi__ring_2} \tag{6}$$

Let $c_2Ecanonical_2Einterp__vl : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ecanonical_2Einterp__vl\ A_27a \in (((A_27a^{(ty_2Elist_2Elist\ ty_2Equote_2Eindex)})^{(ty_2Equote_2Evarmap\ A_27a)})^{(ty_2Esemi__ring_2Esemi__ring\ A_27a)}) \tag{7}$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \tag{8}$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \tag{9}$$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj__o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^{A_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V2t \in 2$

Let $c_2Esemi__ring_2Esemi__ring__SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi__ring_2Esemi__ring__SR1\ A_27a \in (A_27a^{(ty_2Esemi__ring_2Esemi__ring\ A_27a)}) \tag{10}$$

Let $c_2Esemi__ring_2Esemi__ring__SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi__ring_2Esemi__ring__SRM\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Esemi__ring_2Esemi__ring\ A_27a)}) \tag{11}$$

Let $c_2Esemi__ring_2Esemi__ring__SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi__ring_2Esemi__ring__SRP\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Esemi__ring_2Esemi__ring\ A_27a)}) \tag{12}$$

Let $c_2Esemi__ring_2Esemi__ring__SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi__ring_2Esemi__ring__SR0\ A_27a \in (A_27a^{(ty_2Esemi__ring_2Esemi__ring\ A_27a)}) \tag{13}$$

Definition 6 *We define* c_2Esemi__ring_2Eis__semi__ring *to be* $\lambda A_27a : \iota.\lambda V0r \in (ty_2Esemi__ring_2Esemi$

Assume the following.

$$True \tag{14}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0sr \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).(\forall V1vm \in (ty_2Equote_2Evarmap\ A_27a).(\forall V2x \in ty_2Equote_2Eindex.((ap\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Eivl__aux\ A_27a)\ V0sr)\ V1vm)\ V2x)\ (c_2Elist_2ENIL\ ty_2Equote_2Eindex)) = (ap\ (ap\ (c_2Equote_2Evarmap__find\ A_27a)\ V2x)\ V1vm)))))) \wedge (\forall V3sr \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).(\forall V4vm \in (ty_2Equote_2Evarmap\ A_27a).(\forall V5x \in ty_2Equote_2Eindex.(\forall V6x_27 \in ty_2Equote_2Eindex.(\forall V7t_27 \in (ty_2Elist_2Elist\ ty_2Equote_2Eindex).((ap\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Eivl__aux\ A_27a)\ V3sr)\ V4vm)\ V5x)\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Equote_2Eindex)\ V6x_27)\ V7t_27)) = (ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V3sr)\ (ap\ (ap\ (c_2Equote_2Evarmap__find\ A_27a)\ V5x)\ V4vm))\ (ap\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Eivl__aux\ A_27a)\ V3sr)\ V4vm)\ V6x_27)\ V7t_27)))))))))) \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0sr \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).(\forall V1vm \in (ty_2Equote_2Evarmap\ A_27a).((ap\ (ap\ (ap\ (c_2Ecanonical_2Einterp__vl\ A_27a)\ V0sr)\ V1vm)\ (c_2Elist_2ENIL\ ty_2Equote_2Eindex)) = (ap\ (c_2Esemi__ring_2Esemi__ring__SR1\ A_27a)\ V0sr)))) \wedge (\forall V2sr \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).(\forall V3vm \in (ty_2Equote_2Evarmap\ A_27a).(\forall V4x \in ty_2Equote_2Eindex.(\forall V5t \in (ty_2Elist_2Elist\ ty_2Equote_2Eindex).((ap\ (ap\ (ap\ (c_2Ecanonical_2Einterp__vl\ A_27a)\ V2sr)\ V3vm)\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Equote_2Eindex)\ V4x)\ V5t)) = (ap\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Eivl__aux\ A_27a)\ V2sr)\ V3vm)\ V4x)\ V5t))))))) \tag{17}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}).(((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist\ A_27a).(p\ (ap\ V0P\ V3l))))) \tag{18}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).((p\ (ap\ (\mathsf{c_2Esemi__ring_2Eis__semi__ring}\ A_27a)\ V0r)) \Rightarrow (\forall V1n \in A_27a.((ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V0r)\ V1n)\ (ap\ (c_2Esemi__ring_2Esemi__ring__SR1\ A_27a)\ V0r)) = V1n)))) \tag{19}$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0sr \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).((p\ (ap\ (\mathsf{c_2Esemi__ring_2Eis__semi__ring}\ A_27a)\ V0sr)) \Rightarrow (\forall V1vm \in (ty_2Equote_2Evarmap\ A_27a).(\forall V2v \in (ty_2Elist_2Elist\ ty_2Equote_2Eindex).(\forall V3i \in ty_2Equote_2Eindex.((ap\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Eivl__aux\ A_27a)\ V0sr)\ V1vm)\ V3i)\ V2v) = (ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V0sr)\ (ap\ (ap\ (c_2Equote_2Evarmap__find\ A_27a)\ V3i)\ V1vm))\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Einterp__vl\ A_27a)\ V0sr)\ V1vm)\ V2v))))))))$$