thm_2Ecanonical_2Evarlist__merge__ok (TM-PoiKCuBoKf3DM63ithXbuYGSCEhJYNE1D)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj__o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj__o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^{A_27a}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c_2Emin_2E_3D_3D_3E}\ V0t)\ \mathsf{c_2Ebool_2EF}$

Let $ty_2Equote_2Evarmap : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Equote_2Evarmap\ A0) \tag{1}$$

Let $ty_2Equote_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty_2Equote_2Eindex \tag{2}$$

Let $c_2Equote_2Evarmap__find : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Equote_2Evarmap__find\ A_27a \in ((A_27a^{(ty_2Equote_2Evarmap\ A_27a)})^{ty_2Equote_2Eindex}) \tag{3}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{4}$$

Let $ty_2Esemi__ring_2Esemi__ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Esemi__ring_2Esemi__ring\ A0) \tag{5}$$

Let $c_2Ecanonical_2Eivl__aux : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ecanonical_2Eivl__aux\ A_27a \in (((((A_27a^{(ty_2Elist_2Elist\ ty_2Equote_2Eindex)})^{ty_2Equote_2Eindex})^{(ty_2Equote_2Evarmap\ A_27a)})^{(ty_2Esemi__ring_2} \tag{6}$$

Let $c_2Ecanonical_2Einterp__vl : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ecanonical_2Einterp__vl\ A_27a \in (((A_27a^{(ty_2Elist_2Elist\ ty_2Equote_2Eindex)})^{(ty_2Equote_2Evarmap\ A_27a)})^{(ty_2Esemi__ring_2Esemi__ring\ A_27a)}) \tag{7}$$

Let $ty_2EternaryComparisons_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty_2EternaryComparisons_2Eordering \tag{8}$$

Let $c_2EternaryComparisons_2ELESS : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2ELESS \in ty_2EternaryComparisons_2Eordering \tag{9}$$

Let $c_2Equote_2Eindex__compare : \iota$ be given. Assume the following.

$$c_2Equote_2Eindex__compare \in ((ty_2EternaryComparisons_2Eordering^{ty_2Equote_2Eindex})^{ty_2Equote_2E} \tag{10}$$

Definition 8 *We define* c_2Equote_2Eindex__lt *to be* $\lambda V0i1 \in ty_2Equote_2Eindex.\lambda V1i2 \in ty_2Equote_2E$

Let $c_2Esemi__ring_2Esemi__ring__SR1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi__ring_2Esemi__ring__SR1\ A_27a \in (A_27a^{(ty_2Esemi__ring_2Esemi__ring\ A_27a)}) \tag{11}$$

Let $c_2Esemi__ring_2Esemi__ring__SRM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi__ring_2Esemi__ring__SRM\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Esemi__ring_2Esemi__ring\ A_27a)}) \tag{12}$$

Let $c_2Esemi__ring_2Esemi__ring__SRP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi__ring_2Esemi__ring__SRP\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(ty_2Esemi__ring_2Esemi__ring\ A_27a)}) \tag{13}$$

Let $c_2Esemi__ring_2Esemi__ring__SR0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esemi__ring_2Esemi__ring__SR0\ A_27a \in (A_27a^{(ty_2Esemi__ring_2Esemi__ring\ A_27a)}) \tag{14}$$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Definition 10 *We define* c_2Esemi__ring_2Eis__semi__ring *to be* $\lambda A_27a : \iota.\lambda V0r \in (ty_2Esemi__ring_2Esem$

Definition 11 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 12 *We define* c_2Ebool_2ECOND *to be* $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \tag{15}$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \tag{16}$$

Let $c_2EternaryComparisons_2Elist__merge : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2EternaryComparisons_2Elist__merge\ A_27a \in ((((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)})^{((2^{A_27a})^{A_27a})}) \tag{17}$$

Assume the following.

$$True \tag{18}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{21}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{22}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{24}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ A_27a)\ \mathsf{c_2Ebool_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ A_27a)\ \mathsf{c_2Ebool_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{25}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{26}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a.(\forall V5y_27 \in A_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ A_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ A_27a)\ V1Q)\ V3x_27)\ V5y_27))))))))) \tag{27}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0sr \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).(\forall V1vm \in (ty_2Equote_2Evarmap\ A_27a).(\forall V2x \in ty_2Equote_2Eindex.((ap\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Eivl__aux\ A_27a)\ V0sr)\ V1vm)\ V2x)\ (c_2Elist_2ENIL\ ty_2Equote_2Eindex)) = (ap\ (ap\ (c_2Equote_2Evarmap__find\ A_27a)\ V2x)\ V1vm))))) \wedge (\forall V3sr \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).(\forall V4vm \in (ty_2Equote_2Evarmap\ A_27a).(\forall V5x \in ty_2Equote_2Eindex.(\forall V6x_27 \in ty_2Equote_2Eindex.(\forall V7t_27 \in (ty_2Elist_2Elist\ ty_2Equote_2Eindex).((ap\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Eivl__aux\ A_27a)\ V3sr)\ V4vm)\ V5x)\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Equote_2Eindex)\ V6x_27)\ V7t_27)) = (ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V3sr)\ (ap\ (ap\ (c_2Equote_2Evarmap__find\ A_27a)\ V5x)\ V4vm))\ (ap\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Eivl__aux\ A_27a)\ V3sr)\ V4vm)\ V6x_27)\ V7t_27)))))))) \tag{28}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0sr \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).(\forall V1vm \in (ty_2Equote_2Evarmap\ A_27a).((ap\ (ap\ (ap\ (c_2Ecanonical_2Einterp__vl\ A_27a)\ V0sr)\ V1vm)\ (c_2Elist_2ENIL\ ty_2Equote_2Eindex)) = (ap\ (c_2Esemi__ring_2Esemi__ring__SR1\ A_27a)\ V0sr))))) \wedge (\forall V2sr \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).(\forall V3vm \in (ty_2Equote_2Evarmap\ A_27a).(\forall V4x \in ty_2Equote_2Eindex.(\forall V5t \in (ty_2Elist_2Elist\ ty_2Equote_2Eindex).((ap\ (ap\ (ap\ (c_2Ecanonical_2Einterp__vl\ A_27a)\ V2sr)\ V3vm)\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Equote_2Eindex)\ V4x)\ V5t)) = (ap\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Eivl__aux\ A_27a)\ V2sr)\ V3vm)\ V4x)\ V5t))))))) \tag{29}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}).(((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist\ A_27a).(p\ (ap\ V0P\ V3l))))) \tag{30}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).((p\ (ap\ (\mathsf{c_2Esemi__ring_2Eis__semi__ring}\ A_27a)\ V0r)) \Rightarrow (\forall V1n \in A_27a.(\forall V2m \in A_27a.(\forall V3p \in A_27a.((ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V0r)\ V1n)\ (ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V0r)\ V2m)\ V3p)) = (ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V0r)\ (ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V0r)\ V1n)\ V2m))\ V3p))))))) \tag{31}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).((p\ (ap\ (\mathsf{c_2Esemi__ring_2Eis__semi__ring}\ A_27a)\ V0r)) \Rightarrow (\forall V1n \in A_27a.((ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V0r)\ (ap\ (c_2Esemi__ring_2Esemi__ring__SR1\ A_27a)\ V0r))\ V1n) = V1n)))) \tag{32}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).((p\ (ap\ (\mathsf{c_2Esemi__ring_2Eis__semi__ring}\ A_27a)\ V0r)) \Rightarrow (\forall V1n \in A_27a.((ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V0r)\ V1n)\ (ap\ (c_2Esemi__ring_2Esemi__ring__SR1\ A_27a)\ V0r)) = V1n)))) \tag{33}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).((p\ (ap\ (\mathsf{c_2Esemi__ring_2Eis__semi__ring}\ A_27a)\ V0r)) \Rightarrow (\forall V1m \in A_27a.(\forall V2n \in A_27a.(\forall V3p \in A_27a.((ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V0r)\ (ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V0r)\ V1m)\ V2n))\ V3p) = (ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V0r)\ (ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V0r)\ V2n)\ V3p))\ V1m))))))) \tag{34}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V1a__lt \in ((2^{A_27a})^{A_27a}).((ap\ (ap\ (ap\ (c_2EternaryComparisons_2Elist__merge\ A_27a)\ V1a__lt)\ V0l1)\ (c_2Elist_2ENIL\ A_27a)) = V0l1)))) \wedge ((\forall V2v5 \in (ty_2Elist_2Elist\ A_27a).(\forall V3v4 \in A_27a.(\forall V4a__lt \in ((2^{A_27a})^{A_27a}).((ap\ (ap\ (ap\ (c_2EternaryComparisons_2Elist__merge\ A_27a)\ V4a__lt)\ (c_2Elist_2ENIL\ A_27a))\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3v4)\ V2v5)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3v4)\ V2v5))))) \wedge (\forall V5y \in A_27a.(\forall V6x \in A_27a.(\forall V7l2 \in (ty_2Elist_2Elist\ A_27a).(\forall V8l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V9a__lt \in ((2^{A_27a})^{A_27a}).((ap\ (ap\ (ap\ (c_2EternaryComparisons_2Elist__merge\ A_27a)\ V9a__lt)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V6x)\ V8l1))\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V5y)\ V7l2)) = (ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ (ty_2Elist_2Elist\ A_27a))\ (ap\ (ap\ V9a__lt\ V6x)\ V5y))\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V6x)\ (ap\ (ap\ (ap\ (c_2EternaryComparisons_2Elist__merge\ A_27a)\ V9a__lt)\ V8l1)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V5y)\ V7l2)))) (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V5y)\ (ap\ (ap\ (ap\ (c_2EternaryComparisons_2Elist__merge\ A_27a)\ V9a__lt)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V6x)\ V8l1))\ V7l2)))))))))) \tag{35}$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0sr \in (ty_2Esemi__ring_2Esemi__ring\ A_27a).((p\ (ap\ (\mathsf{c_2Esemi__ring_2Eis__semi__ring}\ A_27a)\ V0sr)) \Rightarrow (\forall V1vm \in (ty_2Equote_2Evarmap\ A_27a).(\forall V2x \in (ty_2Elist_2Elist\ ty_2Equote_2Eindex).(\forall V3y \in (ty_2Elist_2Elist\ ty_2Equote_2Eindex).((ap\ (ap\ (ap\ (c_2Ecanonical_2Einterp__vl\ A_27a)\ V0sr)\ V1vm)\ (ap\ (ap\ (ap\ (c_2EternaryComparisons_2Elist__merge\ ty_2Equote_2Eindex)\ \mathsf{c_2Equote_2Eindex__lt})\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c_2Esemi__ring_2Esemi__ring__SRM\ A_27a)\ V0sr)\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Einterp__vl\ A_27a)\ V0sr)\ V1vm)\ V2x))\ (ap\ (ap\ (ap\ (c_2Ecanonical_2Einterp__vl\ A_27a)\ V0sr)\ V1vm)\ V3y)))))))))$$