

thm_2Ecardinal_2EBIJECTIVE__INVERSES (TMK34EvaMdGJZ3gHNNCSGvYguJaYt1dPaLq)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x$

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$
of type ι .

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 8 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C$

Definition 9 We define $c_2Ebool_2E_IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 10 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 11 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (3)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (4)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (6)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\exists V2x \in A_27a. ((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A_27a. (p (ap V1Q V3x)))))) \quad (7)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1f \in (A_27b^{A_27a}). (\forall V2t \in (2^{A_27b}). ((\forall V3y \in A_27b. ((p (ap (ap (c_2Ebool_2EIN A_27b) V3y) V2t)) \Rightarrow (\exists V4x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V4x) V0s)) \wedge ((ap V1f V4x) = V3y)))))) \Leftrightarrow (\exists V5g \in (A_27a^{A_27b}). (\forall V6y \in A_27b. ((p (ap (ap (c_2Ebool_2EIN A_27b) V6y) V2t)) \Rightarrow ((p (ap (ap (c_2Ebool_2EIN A_27a) (ap V5g V6y)) V0s)) \wedge ((ap V1f (ap V5g V6y)) = V6y)))))))))) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1s \in (2^{A_27a}). ((\forall V2x \in A_27a. (\forall V3y \in A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V1s)) \wedge ((p (ap (ap (c_2Ebool_2EIN A_27a) V3y) V1s)) \wedge ((ap V0f V2x) = (ap V0f V3y)))) \Rightarrow (V2x = V3y)))))) \Leftrightarrow (\exists V4g \in (A_27a^{A_27b}). (\forall V5x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V5x) V1s)) \Rightarrow ((ap V4g (ap V0f V5x)) = V5x)))))) \quad (10)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1s \in (2^{A_27a}). (\forall V2t \in \\
& \quad (2^{A_27b}). ((\forall V3x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& \quad V3x)\ V1s)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ (ap\ V0f\ V3x))\ V2t)))) \wedge \\
& \quad (\forall V4y \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V4y)\ V2t)) \Rightarrow \\
& \quad (p\ (ap\ (c_2Ebool_2E_3F_21\ A_27a)\ (\lambda V5x \in A_27a. (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V5x)\ V1s))\ (ap\ (ap\ (c_2Emin_2E_3D \\
& \quad A_27b)\ (ap\ V0f\ V5x))\ V4y)))))) \Leftrightarrow ((\forall V6x \in A_27a. ((p\ (ap\ (\\
& \quad ap\ (c_2Ebool_2EIN\ A_27a)\ V6x)\ V1s)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b) \\
& \quad (ap\ V0f\ V6x))\ V2t))) \wedge ((\forall V7x \in A_27a. (\forall V8y \in A_27a. \\
& \quad (((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V7x)\ V1s)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27a)\ V8y)\ V1s)) \wedge ((ap\ V0f\ V7x) = (ap\ V0f\ V8y))) \Rightarrow (V7x = V8y)))) \wedge \\
& \quad (\forall V9y \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V9y)\ V2t)) \Rightarrow \\
& \quad (\exists V10x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V10x)\ V1s)) \wedge \\
& \quad ((ap\ V0f\ V10x) = V9y))))))))) \\
\end{aligned} \tag{11}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{12}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{15}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False))) \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \\
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \wedge (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (\neg(p \vee V1q)) \vee \neg(p \vee V2r)))) \wedge (((p \vee V1q) \vee \\
& (\neg(p \vee V0p))) \wedge ((p \vee V2r) \vee \neg(p \vee V0p))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \vee (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee \neg(p \vee V1q)) \wedge ((p \vee V0p) \vee \neg(p \vee V2r))) \wedge \\
& ((p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \Rightarrow (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (p \vee V1q)) \wedge (((p \vee V0p) \vee \neg(p \vee V2r))) \wedge (\\
& \neg(p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee V0p) \Leftrightarrow \neg(p \vee V1q)) \Leftrightarrow (((p \vee V0p) \vee \\
& (p \vee V1q)) \wedge (\neg(p \vee V1q) \vee \neg(p \vee V0p))))))
\end{aligned} \tag{21}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow (p \vee V0p))) \tag{22}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow \neg(p \vee V1q))) \tag{23}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V0p))) \tag{24}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V1q))) \tag{25}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \vee V0p))) \Rightarrow (p \vee V0p))) \tag{26}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1s \in (2^{A_27a}). (\forall V2t \in \\
& \quad (2^{A_27b}). (((\forall V3x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& V3x)\ V1s)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ (ap\ V0f\ V3x))\ V2t)))))) \wedge \\
& \quad (\forall V4y \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V4y)\ V2t)) \Rightarrow \\
& (p\ (ap\ (c_2Ebool_2E_3F_21\ A_27a)\ (\lambda V5x \in A_27a. (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V5x)\ V1s))\ (ap\ (ap\ (c_2Emin_2E_3D \\
& \quad A_27b)\ (ap\ V0f\ V5x))\ V4y)))))) \Leftrightarrow ((\forall V6x \in A_27a. ((p\ (ap\ (\\
& \quad ap\ (c_2Ebool_2EIN\ A_27a)\ V6x)\ V1s)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b) \\
& \quad (ap\ V0f\ V6x))\ V2t)))) \wedge (\exists V7g \in (A_27a^{A_27b}). ((\forall V8y \in \\
& A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V8y)\ V2t)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27a)\ (ap\ V7g\ V8y))\ V1s)))) \wedge ((\forall V9y \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27b)\ V9y)\ V2t)) \Rightarrow ((ap\ V0f\ (ap\ V7g\ V9y)) = V9y))) \wedge (\forall V10x \in \\
& A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V10x)\ V1s)) \Rightarrow ((ap\ V7g\ (ap \\
& \quad V0f\ V10x)) = V10x)))))))))
\end{aligned}$$