

thm_2Ecardinal_2EBIJ_functions_agree
(TMZ5Q1rfSs8Kv4w3Dm1pax49K8cvosVYfPi)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_7E` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_7E` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2. V0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t)) (\text{c_2Ebool_2E_7E } V0t)))$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } A))))$

Definition 9 We define `c_2Ebool_2E_IN` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f V0x)))$

Definition 10 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Definition 11 We define `c_2Epred_set_2ESURJ` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27a}).$

Definition 12 We define `c_2Epred_set_2EINJ` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27a}).$

Definition 13 We define `c_2Epred_set_2EBIJ` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27a}).$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A. 27a. (p V0t)) \Leftrightarrow (p V0t))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (3)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (4)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (5)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2) \Rightarrow (p \ V2t3)) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (8)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))))) \quad (9)$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1P_27 \in 2.(\forall V2Q \in 2.(\forall V3Q_27 \in 2.(((p \ V2Q) \Rightarrow ((p \ V0P) \Leftrightarrow (p \ V1P_27))) \wedge ((p \ V1P_27) \Rightarrow ((p \ V2Q) \Leftrightarrow (p \ V3Q_27)))))) \Rightarrow (((p \ V0P) \wedge (p \ V2Q)) \Leftrightarrow ((p \ V1P_27) \wedge (p \ V3Q_27)))))) \quad (10)$$

Theorem 1

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).(\forall V2s \in (2^{A_27a}).(\forall V3t \in (2^{A_27b}).(((p \ (ap \ (ap \ (ap \ (c_2Epred_set_2EBIJ \ A_27a \ A_27b) \ V0f) \ V2s) \ V3t)) \wedge (\forall V4x \in A_27a.((p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \ V4x) \ V2s)) \Rightarrow ((ap \ V0f \ V4x) = (ap \ V1g \ V4x)))))) \Rightarrow (p \ (ap \ (ap \ (ap \ (c_2Epred_set_2EBIJ \ A_27a \ A_27b) \ V1g) \ V2s) \ V3t))))))$$