

thm_2Ecardinal_2ECARD__ADD__CONG
(TMW8rnm5a7W758sxMz2SQaCULdJVLSBMTy7)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a})) (\lambda V1P \in 2. V1P)) (\lambda V2P \in 2. V2P)))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\text{c_2Ebool_2EF } V0t)) (\lambda V1t \in 2. V1t)))$

Definition 7 We define `c_2Ebool_2EIN` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f V0x)))$

Definition 8 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Definition 9 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p (\text{ap } P x)))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } (2^{A-27a})) (\lambda V1P \in 2. V1P))))$

Definition 11 We define `c_2Epred__set_2ESURJ` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27a}). (\text{ap } V1s V0f)$

Definition 12 We define `c_2Epred__set_2EINJ` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27a}). (\text{ap } V1s V0f)$

Definition 13 We define `c_2Epred__set_2EBIJ` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27a}). (\text{ap } V1s V0f)$

Definition 14 We define `c_2Ecardinal_2Ecardeq` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0s1 \in (2^{A-27a}). \lambda V1s2 \in (2^{A-27a}). (\text{ap } V1s2 V0s1)$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Esum_2Esum } A0 A1) \tag{1}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (2)$$

Definition 15 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (3)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (4)$$

Definition 16 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (5)$$

Definition 17 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ V0t1\ V1t2)))$

Definition 19 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ V0s\ V1t)$

Definition 20 We define $c_2Ecardinal_2E_2B_c$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27b}). (ap\ (c_2Ecardinal_2E_2B_c\ A_27a\ A_27b)\ V0s\ V1t)$

Definition 21 We define $c_2Ecardinal_2Ecardleq$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0s1 \in (2^{A_27a}). \lambda V1s2 \in (2^{A_27b}). (ap\ (c_2Ecardinal_2E_2B_c\ A_27a\ A_27b)\ V0s1\ V1s2)$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{9}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{10}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\
& 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27b}). (((p (ap (ap (c_2Ecardinal_2Ecardleq \\
& A_27a\ A_27b)\ V0s)\ V1t)) \wedge (p (ap (ap (c_2Ecardinal_2Ecardleq\ A_27b \\
& A_27a)\ V1t)\ V0s))) \Leftrightarrow (p (ap (ap (c_2Ecardinal_2Ecardleq\ A_27a\ A_27b) \\
& V0s)\ V1t))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0s \in (2^{A_27a}). \\
& (\forall V1s_27 \in (2^{A_27b}). (\forall V2t \in (2^{A_27c}). (\forall V3t_27 \in \\
& (2^{A_27d}). (((p (ap (ap (c_2Ecardinal_2Ecardleq\ A_27a\ A_27b)\ V0s) \\
& V1s_27)) \wedge (p (ap (ap (c_2Ecardinal_2Ecardleq\ A_27c\ A_27d)\ V2t) \\
& V3t_27))) \Rightarrow (p (ap (ap (c_2Ecardinal_2Ecardleq\ (ty_2Esum_2Esum \\
& A_27a\ A_27c)\ (ty_2Esum_2Esum\ A_27b\ A_27d)) (ap (ap (c_2Ecardinal_2E_2B_c \\
& A_27a\ A_27c)\ V0s)\ V2t)) (ap (ap (c_2Ecardinal_2E_2B_c\ A_27b\ A_27d) \\
& V1s_27)\ V3t_27))))))
\end{aligned} \tag{15}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0s \in (2^{A_27a}). \\ & (\forall V1s_27 \in (2^{A_27b}). (\forall V2t \in (2^{A_27c}). (\forall V3t_27 \in \\ & (2^{A_27d}). (((p\ (ap\ (ap\ (c_2Ecardinal_2Ecardeq\ A_27a\ A_27b)\ V0s) \\ V1s_27)) \wedge (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardeq\ A_27c\ A_27d)\ V2t)\ V3t_27))) \Rightarrow \\ & (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardeq\ (ty_2Esum_2Esum\ A_27a\ A_27c) \\ (ty_2Esum_2Esum\ A_27b\ A_27d))\ (ap\ (ap\ (c_2Ecardinal_2E_2B_c \\ A_27a\ A_27c)\ V0s)\ V2t))\ (ap\ (ap\ (c_2Ecardinal_2E_2B_c\ A_27b\ A_27d) \\ V1s_27)\ V3t_27)))))))))) \end{aligned}$$