

thm_2Ecardinal_2ECARD__DISJOINT__UNION
(TMKC7FseFRXGoW7gm69TYnogMZ3kKoVJZwF)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$
of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t)))$

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p (ap P x))$)
of type $\iota \Rightarrow \iota$.

Definition 7 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))))$

Definition 8 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C A_27a P)))$

Definition 9 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 10 We define $c_2Epred_set_2ESURJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a}).$

Definition 11 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a}).$

Definition 12 We define $c_2Epred_set_2EBIJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a}).$

Definition 13 We define $c_2Ecardinal_2Ecardeq$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s1 \in (2^{A_27a}).\lambda V1s2 \in (2^{A_27a}).$

Definition 14 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 15 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (1)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (2)$$

Definition 16 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (3)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (4)$$

Definition 17 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (5)$$

Definition 18 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ V0t1\ V1t2))$

Definition 20 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ V0s\ V1t)$

Definition 21 We define $c_2Ecardinal_2E_2B_c$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27b}).(ap\ (c_2Ecardinal_2E_2B_c\ A_27a\ A_27b)\ V0s\ V1t)$

Definition 22 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 23 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ V0s\ V1t)$

Definition 24 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (c_2Ebool_2EF)\ V0t1\ V2t2))))$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee (\neg(p \ V0t)))) \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \wedge ((p \ V1t2) \wedge (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \wedge (p \ V2t3)))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (15)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (16)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\neg(\forall V1x \in A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a. (\neg(p (ap V0P V2x))))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\neg(\exists V1x \in A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a. (\neg(p (ap V0P V2x))))) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in (2^{A.27a}). ((\exists V2x \in A.27a. ((p (ap V0P V2x)) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x))) \vee (\exists V4x \in A.27a. (p (ap V1Q V4x))))))) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((p V0P) \vee (\exists V2x \in A.27a. (p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a. ((p V0P) \vee (p (ap V1Q V3x)))))) \quad (23)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\exists V2x \in A.27a. ((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a. (p (ap V1Q V3x))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. ((\neg((p V0A) \Rightarrow (p V1B))) \Leftrightarrow ((p V0A) \wedge (\neg(p V1B))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ & \quad \forall V0b \in 2.(\forall V1f \in (A_27b^{A_27a}).(\forall V2g \in (A_27b^{A_27a}). \\ & \quad (\forall V3x \in A_27a.((ap (ap (ap (ap (c_2Ebool_2ECOND (A_27b^{A_27a})) \\ & \quad V0b) V1f) V2g) V3x) = (ap (ap (ap (c_2Ebool_2ECOND A_27b) V0b) (ap \\ & \quad V1f V3x)) (ap V2g V3x)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}).(\forall V1b \in 2.(\forall V2x \in A_27a. \\ & \quad (\forall V3y \in A_27a.((ap V0f (ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & \quad V1b) V2x) V3y)) = (ap (ap (ap (c_2Ebool_2ECOND A_27b) V1b) (ap V0f \\ & \quad V2x)) (ap V0f V3y)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((p (ap \\ & \quad (c_2Ebool_2E_3F_21 \ A_27a) (\lambda V1x \in A_27a.(ap V0P V1x)))) \Leftrightarrow ((\\ & \quad \exists V2x \in A_27a.(p (ap V0P V2x))) \wedge (\forall V3x \in A_27a.(\forall V4y \in \\ & \quad A_27a.(((p (ap V0P V3x)) \wedge (p (ap V0P V4y))) \Rightarrow (V3x = V4y)))))) \end{aligned} \quad (32)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (33)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0s \in (2^{A.27a}).(\forall V1t \in (2^{A.27b}).((p\ (ap\ (ap\ (c.2Ecardinal.2Ecardeq \\
& \quad A.27a\ A.27b)\ V0s)\ V1t))) \Leftrightarrow (\exists V2f \in (A.27b^{A.27a}).(\forall V3x \in \\
& A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V3x)\ V0s)) \Rightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& \quad A.27b)\ (ap\ V2f\ V3x))\ V1t)))) \wedge (\forall V4y \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& \quad A.27b)\ V4y)\ V1t)) \Rightarrow (p\ (ap\ (c.2Ebool.2E.3F.21\ A.27a)\ (\lambda V5x \in A.27a. \\
& \quad (ap\ (ap\ c.2Ebool.2E.2F.5C\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V5x)\ V0s)) \\
& \quad (ap\ (ap\ (c.2Emin.2E.3D\ A.27b)\ (ap\ V2f\ V5x))\ V4y)))))))))) \\
& \hspace{15em} (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0s \in (2^{A.27a}).(\forall V1t \in (2^{A.27b}).(\forall V2x \in \\
& \quad A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (ty.2Esum.2Esum\ A.27a\ A.27b)) \\
& \quad (ap\ (c.2Esum.2EINL\ A.27a\ A.27b)\ V2x))\ (ap\ (ap\ (c.2Ecardinal.2E.2B._c \\
& \quad A.27a\ A.27b)\ V0s)\ V1t))) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)))) \wedge \\
& \quad (\forall V3y \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (ty.2Esum.2Esum \\
& \quad A.27a\ A.27b))\ (ap\ (c.2Esum.2EINR\ A.27a\ A.27b)\ V3y))\ (ap\ (ap\ (c.2Ecardinal.2E.2B._c \\
& \quad A.27a\ A.27b)\ V0s)\ V1t))) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27b)\ V3y)\ V1t)))))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\
& (2^{A.27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& \quad A.27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V1t)))))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\neg (p\ (ap\ (ap \\
& \quad (c.2Ebool.2EIN\ A.27a)\ V0x)\ (c.2Epred._set.2EEMPTY\ A.27a)))) \\
& \hspace{15em} (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\
& (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\
& \quad V2x)\ (ap\ (ap\ (c.2Epred._set.2EUNION\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (38) \\
& \quad (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& \quad A.27a)\ V2x)\ V1t))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\
& (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\
& \quad V2x)\ (ap\ (ap\ (c.2Epred._set.2EINTER\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (39) \\
& \quad (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& \quad A.27a)\ V2x)\ V1t))))))
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (49)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (\forall V3s \in \\
& 2. (((p \ V0p) \Leftrightarrow (p \ (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ 2) \ V1q) \ V2r) \ V3s)))) \Leftrightarrow \\
& (((p \ V0p) \vee ((p \ V1q) \vee (\neg(p \ V3s)))) \wedge (((p \ V0p) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V1q)))) \wedge \\
& (((p \ V0p) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V3s)))) \wedge (((\neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p)))) \wedge ((p \ V1q) \vee ((p \ V3s) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\
& \quad \forall V0P \in (2^{(ty_2Esum_2Esum \ A_27a \ A_27b)}). ((\forall V1s \in \\
& \quad (ty_2Esum_2Esum \ A_27a \ A_27b). (p \ (ap \ V0P \ V1s))) \Leftrightarrow ((\forall V2x \in \\
& \quad A_27a. (p \ (ap \ V0P \ (ap \ (c_2Esum_2EINL \ A_27a \ A_27b) \ V2x)))) \wedge (\forall V3y \in \\
& \quad A_27b. (p \ (ap \ V0P \ (ap \ (c_2Esum_2EINR \ A_27a \ A_27b) \ V3y))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow \forall A_27c. \\
& nonempty \ A_27c \Rightarrow \forall A_27d.nonempty \ A_27d \Rightarrow \forall A_27e.nonempty \\
& A_27e \Rightarrow \forall A_27f.nonempty \ A_27f \Rightarrow \forall A_27g.nonempty \ A_27g \Rightarrow \\
& \quad \forall A_27h.nonempty \ A_27h \Rightarrow (\forall V0P \in 2. ((\forall V1x \in \\
& \quad A_27a. (\forall V2y \in A_27b. (\forall V3z \in A_27a. (((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \\
& \quad (ty_2Esum_2Esum \ A_27b \ A_27a) \ V0P) \ (ap \ (c_2Esum_2EINR \ A_27b \ A_27a) \\
& \quad V1x)) \ (ap \ (c_2Esum_2EINL \ A_27b \ A_27a) \ V2y)) = (ap \ (c_2Esum_2EINR \\
& \quad A_27b \ A_27a) \ V3z)) \Leftrightarrow ((p \ V0P) \wedge (V3z = V1x)))))) \wedge ((\forall V4x \in A_27c. \\
& \quad (\forall V5y \in A_27d. (\forall V6z \in A_27d. (((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \\
& \quad (ty_2Esum_2Esum \ A_27d \ A_27c) \ V0P) \ (ap \ (c_2Esum_2EINR \ A_27d \ A_27c) \\
& \quad V4x)) \ (ap \ (c_2Esum_2EINL \ A_27d \ A_27c) \ V5y)) = (ap \ (c_2Esum_2EINL \\
& \quad A_27d \ A_27c) \ V6z)) \Leftrightarrow ((\neg(p \ V0P)) \wedge (V6z = V5y)))))) \wedge ((\forall V7x \in \\
& \quad A_27e. (\forall V8y \in A_27f. (\forall V9z \in A_27e. (((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \\
& \quad (ty_2Esum_2Esum \ A_27e \ A_27f) \ V0P) \ (ap \ (c_2Esum_2EINL \ A_27e \ A_27f) \\
& \quad V7x)) \ (ap \ (c_2Esum_2EINR \ A_27e \ A_27f) \ V8y)) = (ap \ (c_2Esum_2EINL \\
& \quad A_27e \ A_27f) \ V9z)) \Leftrightarrow ((p \ V0P) \wedge (V9z = V7x)))))) \wedge ((\forall V10x \in A_27g. \\
& \quad (\forall V11y \in A_27h. (\forall V12z \in A_27h. (((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \\
& \quad (ty_2Esum_2Esum \ A_27g \ A_27h) \ V0P) \ (ap \ (c_2Esum_2EINL \ A_27g \ A_27h) \\
& \quad V10x)) \ (ap \ (c_2Esum_2EINR \ A_27g \ A_27h) \ V11y)) = (ap \ (c_2Esum_2EINR \\
& \quad A_27g \ A_27h) \ V12z)) \Leftrightarrow ((\neg(p \ V0P)) \wedge (V12z = V11y)))))))))
\end{aligned} \tag{52}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& \quad (2^{A_27a}). (((ap \ (ap \ (c_2Epred_set_2EINTER \ A_27a) \ V0s) \ V1t) = \\
& \quad (c_2Epred_set_2EEMPTY \ A_27a)) \Rightarrow (p \ (ap \ (ap \ (c_2Ecardinal_2Ecardeq \\
& \quad A_27a \ (ty_2Esum_2Esum \ A_27a \ A_27a) \ (ap \ (ap \ (c_2Epred_set_2EUNION \\
& \quad A_27a) \ V0s) \ V1t)) \ (ap \ (ap \ (c_2Ecardinal_2E_2B_c \ A_27a \ A_27a) \ V0s) \\
& \quad V1t))))))
\end{aligned}$$